4D Fourier Transform

$$\frac{e^{i\omega' t}}{\sqrt{2\pi}} \frac{e^{ikz}}{\sqrt{2\pi}} \frac{\sin(m\pi\theta / \alpha)}{\sqrt{\alpha / 2}} \frac{\phi_{mj}(r)}{N_{mj}}$$

Fourier Domain T

$$-(\omega')^2\tilde{T} - k^2\tilde{T} - \omega_{mj}{}^2\tilde{T} = 2\,\frac{\sin(m\pi\theta'/\alpha)}{\sqrt{\alpha/2}}\,\frac{\phi_{mj}(r')}{N_{mj}}$$

Prenormalized Radial Eigenfunction

$$\phi_{mj} = Y_\nu(\omega_{mj} a) J_\nu(\omega_{mj} r) - J_\nu(\omega_{mj} a) Y_\nu(\omega_{mj} r)$$

Fourier Domain T

$$-(\omega')^2\tilde{T} - k^2\tilde{T} - \omega_{mj}{}^2\tilde{T} = 2\frac{\sin(m\pi\theta'/\alpha)}{\sqrt{\alpha/2}}\frac{\phi_{mj}(r')}{N_{mj}}$$

Inverse Transform T

$$\bar{T} = \frac{-2}{\pi\alpha}\int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} dk\, e^{i\omega' t} e^{ikz} \sum_{m=1}^{\infty} \sin\left(m\pi\theta / \alpha\right) \sin\left(m\pi\theta' / \alpha\right)$$

$$\sum_{j=1}^{\infty} \frac{1}{N_{mj}^{\;\;2}} \frac{\phi_{mj}\left(r\right)\phi_{mj}\left(r'\right)}{\left(\omega'\right)^2 + \omega_{mj}^{\;\;2} + k^2}$$

Approach 2 Radial

$$\sum_{j=1}^{\infty} \frac{1}{N_{mj}{}^{2}} \frac{\phi_{mj}(r)\phi_{mj}(r')}{(\omega')^{2} + \omega_{mj}{}^{2} + k^{2}} \rightarrow$$

$$I_{\nu}(\kappa\rho_{<})K_{\nu}(\kappa\rho_{>}) - K_{\nu}(\kappa\rho)K_{\nu}(\kappa\rho')\frac{I_{\nu}(\kappa\alpha)}{K_{\nu}(\kappa\alpha)}$$

Fourier Polar Coordinates

$$\omega' = \kappa \cos(c)$$

$$k = \kappa \sin(c)$$

$$t = \delta \cos(\phi)$$

$$z = \delta \sin(\phi)$$

Asymptotic Expansion

$$\begin{aligned}
\overline{T}_1 &= -\frac{2}{\alpha}\mathcal{A}\int_0^\infty \kappa\, d\kappa\, \frac{J_0(\kappa\delta)}{\nu\sqrt{1+z^2}} \\
&= -\frac{2}{\alpha\rho^2}\mathcal{A}\int_0^\infty z\, dz\, \frac{\nu J_0\left(\frac{\nu\delta z}{\rho}\right)}{\sqrt{1+z^2}} \\
&= -\frac{2}{\alpha\rho\delta}\mathcal{A}e^{-\nu\delta/\rho}
\end{aligned}$$

First Term

$$\begin{aligned}
\overline{T}_1 &= -\frac{2}{\alpha}\mathcal{A}\int_0^\infty \kappa\, d\kappa\, \frac{J_0(\kappa\delta)}{\nu\sqrt{1+z^2}} \\
&= -\frac{2}{\alpha\rho^2}\mathcal{A}\int_0^\infty z\, dz\, \frac{\nu J_0\left(\frac{\nu\delta z}{\rho}\right)}{\sqrt{1+z^2}} \\
&= -\frac{2}{\alpha\rho\delta}\mathcal{A}e^{-\nu\delta/\rho}
\end{aligned}$$

Dowker Manifold

$$\mathcal{A} \to \frac{\alpha}{\pi} \int_0^\infty d\nu \, \sin \nu\theta \sin \nu\theta'$$

$$\begin{aligned}
\overline{T}_1 &= -\frac{2}{\pi^2 \rho\theta\delta} \int_0^\infty \sin^2 x \, e^{-\delta x / \rho\theta} \, dx \\
&= -\frac{4}{\pi^2 \delta^2} \left(4 + \frac{\delta^2}{\rho^2 \theta^2} \right)^{-1} \\
&\sim -\frac{1}{\pi^2 \delta^2} + \frac{1}{4\pi^2 \rho^2 \theta^2} + \cdots .
\end{aligned}$$

Second Term Intro

$$I_\nu(\kappa\rho_<)K_\nu(\kappa\rho_>) - K_\nu(\kappa\rho)K_\nu(\kappa\rho')\frac{I_\nu(\kappa\alpha)}{K_\nu(\kappa\alpha)}$$

Second Term

$$\overline{T}_2 = \frac{2}{\alpha}\mathcal{A}\int_0^\infty \kappa\, d\kappa\, \frac{J_0(\kappa\delta)e^{2\nu(\eta_a-\eta_\rho)}}{\nu\sqrt{1+z(\rho)^2}}[1+\cdots]$$

$$\eta_a - \eta_\rho \sim -\frac{1}{a}\sqrt{1+\left(\frac{ka}{\nu}\right)^2}\,(\rho - a)$$

$$e^{2\nu(\eta_a-\eta_\rho)} \sim 1 - \frac{2\nu}{a}\sqrt{1+\left(\frac{ka}{\nu}\right)^2}\,(\rho - a)$$

Jo Terminus

$$\bar{T} = \frac{-2}{\pi\alpha}\int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} dk\, e^{i\omega' t + ikz} A g_{\nu\kappa}(\rho, \rho') \rightarrow$$

$$\bar{T} = -\frac{4}{\alpha} A \int_{0}^{\infty} \kappa d\kappa J_0(\kappa\delta) g_{\nu\kappa}(\rho, \rho')$$