Entropy of symbolic dynamical systems and of formal languages

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We recall the definition of *entropy of a finite graph* and present a new proof ([8], based on ideas of Gromov [7] and Perron-Frobenius free) of the following fact: *Suppose that $G = (V, E)$ is a finite irreducible di-graph; let $e \in E$ be an edge and denote by $G_e = (V, E \setminus \{e\})$ the graph obtained by removing the edge e. Then $ent(G_e) < ent(G)$ (strictly!).*

One derives applications in the settings of symbolic dynamics and in that of formal languages.

More precisely, we deduce the Hedlund-Coven-Paul theorem for finite (and more generally for sofic) shifts. This is related to the Garden of Eden type theorems for cellular automata over amenable structures [2, 7, 6] (see also [1]).

As remarked by W. Krieger, sofic shifts are close to *regular languages* and a similar statement can be presented (indeed this is known but the proof is always based on Perron-Frobenius theory): *Let L be an irreducible regular language. Let w be a word or subword of L and denote by L_w the sublanguage of L obtained by forbidding the word w. Then $ent(L_w) < ent(L)$ (strictly!).* When applied to the language of (normal forms for) free groups one can derive, together with some additional aysmptotic-group-theoretical arguments, a new proof of the *hopfianity* of finitely gererated free groups (recall that a group is *hopfian* if it is *not* isomorphic to a proper quotient) [3, 4].

In Chomsky's hierarchy, above the regular languages one finds the *context-free languages*. In [5] a definition of *ergodicity* (or equivalently of irreducibility) is presented for these languages and an analogue – of the theorem above on the entropy of regular languages – is proved for context-free non-linear languages. The proof is rather technical and involves several ideas combining Perron-Frobenius theory with random walks methods (à la Lalley) and with the algebraic theory of these languages à la Chomsky-Schützenberger.

References

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