Texas Geometry and Topology Conference

This is a report on the presentations at the 34th meeting of the Texas Geometry and Topology Conference at The University of Texas at Austin, September 30-October 2, 2005. This conference was partially supported by National Science Foundation Grant DMS-0306628 and The University of Texas. Speakers reported on recent research. For this report, speakers have provided synopses of their talks together with broader discussions of the significance and context of their results.

Meeting 34. The University of Texas, September 30-October 2, 2005

David Ben-Zvi, The University of Texas, *Langlands duality and representations of real groups*

We will discuss the idea of a dual group, starting from abelian (Fourier/Pontrjagin/Cartier) duality, through the parametrization of representations of real semisimple Lie groups and towards the Langlands duality for representations of groups over a local field. We will conclude by discussing work with David Nadler (Northwestern) on the relation between real Lie groups and the geometric Langlands duality.

George Daskalopoulos, Brown University, *Harmonic maps on singular domains and geometric rigidity*

I presented the theory of harmonic maps from flat 2-complexes into mehri spaces of nonposition curvature. Existence and regularity questions were addressed. In particular, if the domain complex satisfied certain geometric and combinatorial conditions, then the regularity together with a Bochner formula lead to rigidity and vanishing theorem.

John Etnyre, Georgia Institute of Technology, *Contact geometry and open book genus*

In this talk I discuss Grioux's correspondence between open book decompositions of 3-manifolds and contact structures. This discussion naturally leads to a new invariant of contact structures: the support genus. This is the minimal genus of an open book that supports a given contact structures. I will present several theorems that indicate the support genus sees subtle properties of the contact structure. For example if the support genus is larger than 0, then the contact structure is tight. Finally I will show that the support genus can be larger than 0, and that if it is 0 then this can be useful in understanding symplectic fillings of the contact structures.

Nancy Hingston, The College of New Jersey, *Periodic solutions of Hamilton's equations on tori*

Let the torus T^{2n} carry the standard symplectic structure, and a Hamiltonian function H of period 1 in the time variable. By the Arnold Conjecture, proved for the torus by Conley and Zehnder, the Hamiltonian flow has at least $2n + 1$ orbits of period 1. Conley and Zehnder also proved, under the additional assumption that all period 1 orbits are nondegenerate: If there are only finitely many orbits of period 1, then there are orbits of arbitrarily large minimal (integer) period. We prove this statement also holds in the degenerate case. Thus there are always infinitely many orbits of integer period. This settles a conjecture of Conley for the torus; this conjecture is still open for other compact

Sheldon Katz, University of Illinois at Urbana-Champaign, *Donaldson-Thomas invariants of complex projective threefolds*

In recent years, there has been much interest in invariants of complex projective threefolds inspired by string theory: Gromov-Witten invariants, Gopakumar-Vafa invariants (not yet rigorously defined mathematically), and most recently, Donaldson-Thomas invariants. All of these invariants are conjecturally related to each other.

In this talk, I explain how Donaldson-Thomas invariants have evolved from holomorphic Chern-Simons gauge theory and sketch some foundations of the theory of Donaldson-Thomas invariants. These invariants are applied towards the definition of Gopakumar-Vafa invariants. It is proven that this gives the right answer in cases including low degree invariants of local toric Calabi-Yau threefolds and certain local contractible curves.

Rafe Mazzeo, Stanford University, *Some linear and nonlinear elliptic problems in higher rank geometries*

Noncompact symmetric spaces of rank greater than one occupy a ground somewhere between Euclidean and hyperbolic space. The different types of global analytic behaviour which occur in these extreme cases is well-known. In this talk I will report on some recent progress in the development of microlocal techniques to study global analytic problems on a class of spaces with asymptotically symmetric structures at infinity. Part of this, joint with Vasy, concerns refined aspects of scattering theory on these spaces. But in this talk I shall report on recent results with Biquard concerning the possibility of deforming globally symmetric spaces in the class of Einstein metrics.

Paul Seidel, University of Chicago, *Khovanov meets Ozsvath-Szabo*

This is joint work with Ivan Smith. Khovanov cohomology is a bigraded group $Kh^{*,*}(\kappa)$ associated to an oriented link $\kappa \subset \mathbb{R}^3$. It is defined combinatorially, and related to the Jones polynomial. Ozsvath-Szabo theory provides a similar invariant $\widehat{HF}^*(S^3_\kappa \# S^1 \times S^2)$, obtained by forming the double branched cover S^3_κ along κ, adding a trivial $S^1 \times S^2$ summand, and looking at its Heegaard Floer theory.

We start with a geometric version $Kh^*_{\text{symp}}(\kappa)$ of Khovanov cohomology, defined (following Rasmuller) in terms of Hilbert schemes of ALE spaces. We apply localization techniques from equivariant cohomology to the associated Floer cohomology groups, and obtain a spectral sequence connecting $Kh^*_{\text{symp}}(\kappa)$ to $\widehat{HF}^*(S^3_\kappa \# S^1 \times S^2)$. One can also extract additional invariants from this relationship.

Dennis Sullivan, SUNY Stony Brook, *The free loop space of a manifold and the moduli space of Riemann surfaces*

Basic operations in the loop space were constructed in "String Topology" with Moira Chas. The full development of these operations could be described as follows:

> Chains on the space of string diagrams (see below) acts on the loop space of any manifold

Now we extend this using two ideas. The first point is that the ideas of space of string diagrams corresponds exactly to pairs (Riemann surface with punctures, proper harmonic functions) so that the harmonic function has only one critical level. The second idea is to introduce a propagation to extend the original construction to general pairs (surface, function) with several critical levels.

A propagation is a $(d-1)$ form on $M \times M$ ($d = \dim M$) providing a homology supported near the diagonal between the diagonal and a third class of the diagonal.

The result that emerges is an action of the chains on the entire moduli space on the chains of the free loop space of any manifold.