Texas Geometry and Topology Conference

This is a report on the presentations at the 56th meeting of the Texas Geometry and Topology Conference at Texas A&M University on November 6-8, 2016. This conference was partially supported by National Science Foundation Grant DMS-1510060, and Texas A&M University. Speakers reported on recent research. All plenary speakers provided abstracts. Plenary speakers were encouraged to offer in their abstracts slightly broader discussions of the significance and context of their results.

Meeting 56. Texas A&M University, November 4-6, 2016

Anna Marie Bohmann, Vanderbilt, *Constructing equiviant cohomology*

Equivariant cohomology theories that are cohomology theories incorporate a group action on spaces. These types of cohomology theories are increasingly important in algebraic topology but can be difficult to understand or construct. In recent work, Angélica Osorno and I have developed a construction for building them out of purely algebraic data based on symmetric monoidal categories. Our method is philosophically similar to classical work of Segal on building nonequivariant cohomology theories. In this talk I will discuss this work, and as well as an extension to the more general world of Waldhausen categories. Our new construction is more flexible and is designed to be suitable for equivariant algebraic K-theory constructions.

Robert Hardt, Rice University, *Some analytic properties of singular spaces described by special classes of chains and cochains*

Classes of chains and cochains may reveal geometric as well as topological properties of metric spaces. For analytic properties, flat chains introduced by Whitney in 1957, had great influence on the development of geometric measure theory and higher dimensional calculus of variations. Normal chains, which have finite mass and boundary mass, in metric spaces and, for real coefficients, dual cochains, called charges, have been studied recently in joint works with T. DePauw and W. Pfeffer. Metric spaces satisfying a linear isoperimetric property enjoy a full duality between the normal chain homology and charge cohomology. Recent work with DePauw has established this property for real algebraic and analytic spaces, in all dimensions and codimensions. For these spaces, it is handy to work with the much smaller classes of (semi) algebraic or (sub) analytic chains. Such chains are also useful in another work, with P. dosSantos, J. Lewis, and P. Lima-Filho, on an explicit construction of cycle maps from motivic cohomology of Voevodsky correspondences to ordinary equivariant singular cohomology.

Vaughn Jones, Vanderbilt, *Knots and the Thompson groups*

We give a construction of a knot or link in 3-space from an element of the Thompson group and show that all knots and links arise in this way. This establishes the Thompson group as a knot constructor as good as the braid groups. Moreover multiplication in the Thompson group is relatively close to the connect sum of knots and links!

Vaughn Jones, Vanderbilt, *On some unitary representations of the Thompson groups*

We give a simple categorical way to construct actions of the Thompson groups. In particular this gives representations of the Thompson groups in $\mathrm{Diff}(S^1)$ and as unitaries on Hilbert space in many ways.

Niky Kamran, McGill, *Lorentzian Einstein metrics with prescribed conformal infininity*

A classical theorem Graham and Lee states that for any metric on S^n that is $C^{2,\alpha}$-close to the round metric, the ball B^{n+1} admits an Einstein metric that is $C^{2,\alpha}$-close to the Poincaré metric and such that its restriction to the boundary corresponds (after conformal rescaling) to the given metric on S^n. Our talk is concerned with the Lorentzian version of this problem, where the ball B^{n+1} is replaced by a open cylinder $R \times B^n$, endowed with the anti-de Sitter metric. This problem is relevant to the formulation of a fully Lorentzian version of the AdS/CFT correspondence. We will present a local existence theorem that answers this question, and outline the main ideas of the proof. This is joint work with Alberto Enciso (ICMAT, Madrid).

Shrawan Kumar, UNC Chapel Hill, *Verlinde dimension formula for the space of conformal blocks and the moduli of G-bundles*

Let G be a simply-connected complex semisimple algebraic group and let C be a smooth projective curve of any genus. E. Verlinde conjectured a remarkable formula to calculate the dimension of the space of generalized theta functions, which I will define in the lecture. These generalized theta functions are 'non-abelian' analog of the classical theta functions. This space is also identified with the space of conformal blocks arising in Conformal Field Theory, which is by definition the space of coinvariants in integrable highest weight modules of affine Lie algebras. Various works notably by Tsuchiya-Ueno-Yamada, Kumar-Narasimhan-Ramanathan, Faltings, Beauville-Laszlo, Sorger and Teleman culminated into a proof of the Verlinde formula.

The main aim of this talk is to give an overview of this formula. Though we will not have time to go into any detailed proofs, the proof requires techniques from algebraic geometry, geometric invariant theory, representation theory, and topology.

Shrawan Kumar, UNC Chapel Hill, *Representation ring of Levi subgroups versus cohomology ring of flag varieties*

Recall the classical result that the cup product structure constants for the singular cohomology with integral coefficients of the Grassmannian of r-planes coincide with the Littlewood-Richardson tensor product structure constants for $\mathbf{GL}(r)$. Specifically, the result asserts that there is an explicit ring homomorphism $\phi : (\mathbf{GL}(r)) \to H^*((r,n))$, where (r,n) denotes the Grassmannian of r-planes in $\mathbb{C}^n$ and $(Ł(r))$ denotes the polynomial representation ring of $\mathbf{GL}(r)$.

This talk seeks to achieve one possible generalization of this classical result for $\mathbf{GL}(r)$ and the Grassmannian (r,n) to the Levi subgroups of any reductive group G and the corresponding flag varieties.

Rafe Mazzeo, Stanford University, *QFB geometry and the Hitchin moduli space*

I describe some recent progress on understanding the global geometry of the natural Weil-Petersson metric on the moduli space of stable Higgs bundles. This connects with some interesting physical predictions. This is joint work with Swoboda, Weiss and Witt.

Emmy Murphy, MIT, *Legendrian knots and affine varieties*

A smooth complex affine algebraic surface is a special case of symplectic 4-manifold. Furthermore it admits a Weinstein handle decomposition, which means that it can be completely described (up to symplectomorphism) by a Legendrian link in the standard contact $k\#(S^1 \times S^2)$. We'll discuss how to translate in practice from a set of polynomial equations in C^N to an explicit front of a Legendrian link, making a stop in between in the world of Lefschetz fibrations. We'll then discuss why this is useful, particularly giving applications to computations of Fukaya categories and mirror symmetry. If there's some extra time we'll talk about the higher dimensional case, particularly how tools from the world of high dimensional contact flexibility give new results in other directions.

Thomas Schick, U. Goettingen, *Obstructions to positive scalar curvature via submanifolds of different codimension*

We want to discuss a collection of results around the following Question: Given a smooth compact manifold M without boundary, does M admit a Riemannian metric of positive scalar curvature?

We focus on the case of spin manifolds. The spin structure, together with a chosen Riemannian metric, allows to construct a specific geometric differential operator, called Dirac operator. If the metric has positive scalar curvature, then 0 is not in the spectrum of this operator; this in turn implies that a topological invariant, the index, vanishes.

We use a refined version, acting on sections of a bundle of modules over a C^*-algebra; and then the index takes values in the K-theory of this algebra. This index is the image under the Baum-Connes assembly map of a topological object, the K-theoretic fundamental class.

The talk will present results of the following type:

If M has a submanifold N of codimension k whose Dirac operator has non-trivial index, what conditions imply that M does not admit a metric of positive scalar curvature? How is this related to the Baum-Connes assembly map?

We will present previous results of Zeidler ($k = 1$), Hanke-Pape-S. ($k = 2$), Engel and new generalizations. Moreover, we will show how these results fit in the context of the Baum-Connes assembly maps for the manifold and the submanifold.