

Name _____

MATH 172 Honors Exam 2 Spring 2024

Section 200 P. Yasskin

Points indicated. Part credit possible. Show all work.

| | | | | | |
|-------|----|---|-------|----|-----|
| 1 | /? | 5 | /10 | 9 | /4 |
| 2 | /8 | 6 | /10 | 10 | /8 |
| 3 | /8 | 7 | /10 | 11 | /10 |
| 4 | /8 | 8 | /10 | 12 | /10 |
| Total | | | /96+? | | |

1. (? points) Circle each term in the general partial fraction expansion for $p(x) = \frac{5x}{(x-2)(x^4-16)}$.

(2 points for each correct term. -2 points for each incorrect term.)

$$\frac{A}{x+2} \quad \frac{C}{x-2} \quad \frac{Ex+F}{x^2-4} \quad \frac{Ix+J}{x^2+4}$$
$$\frac{B}{(x+2)^2} \quad \frac{D}{(x-2)^2} \quad \frac{Gx+H}{(x^2-4)^2} \quad \frac{Kx+L}{(x^2+4)^2}$$

2. (8 points) Find the coefficients in the partial fraction expansion

$$\frac{48x^2}{(x^4-81)} = \frac{A}{x-3} + \frac{B}{x+3} + \frac{Cx+D}{x^2+9}$$

Write the final expansion.

3. (8 points) Given that $\frac{3x^3 + 18x^2 - 81x + 162}{x(x^2 + 9)(x - 3)^2} = \frac{2}{x} + \frac{3}{(x - 3)^2} + \frac{-2x}{x^2 + 9}$

compute $I = \int \frac{3x^3 + 18x^2 - 81x + 162}{x(x^2 + 9)(x - 3)^2} dx.$

4. (8 points) Compute the improper integral $I = \int_0^1 \frac{2x}{\sqrt{1 - x^2}} dx.$

5. (10 points) The area between $x = 1 - (y - 1)^2$ and the y -axis is rotated about the x -axis. Find the volume swept out.

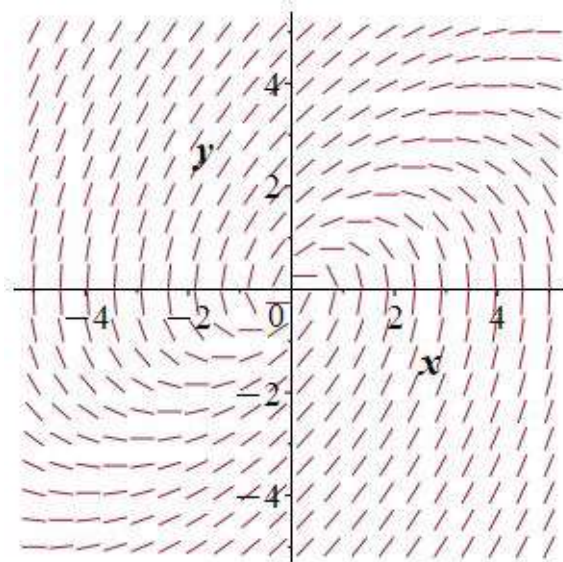
6. (10 points) The area between $x = 1 - (y - 1)^2$ and the y -axis is rotated about the y -axis. Find the volume swept out.

7. (10 points) Solve the initial value problem $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^3$ with $y(1) = 2$.

8. (10 points) Solve the initial value problem $3\frac{dy}{dx} + \frac{2x}{y^2} = \frac{1}{y^2}$ with $y(2) = 1$.

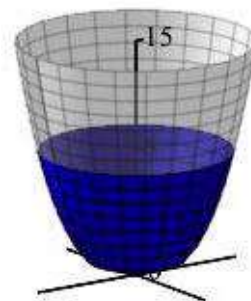
Give the explicit solution.

9. (4 points) At the right is the slope field of the differential equation $\frac{dy}{dx} = \frac{y-x}{y}$. On this plot, roughly sketch the solution curve satisfying the initial condition $y(1) = 4$.



10. (8 points) The area between the curve $y = \frac{1}{x^p}$ and the x -axis for $1 \leq x \leq \infty$ is rotated about the x -axis, sweeping out a volume. For which values of p is the volume finite? Be sure to check the border line case.

11. (10 points) A water tank is formed by rotating the curve $y = x^3$ for $y \leq 15$ meters about the y -axis. It is filled to a depth of 8 meters. Find the volume of water in the tank.
HINT: Use horizontal slices.



12. (10 points) For the water tank described in the previous problem, find the work done to pump the water out the top of the tank.
Give your answer as a multiple of δg where δ is the density of water and g is the acceleration of gravity.