Name_____

MATH 221   Exam 1, Version B   Spring 2024

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  Multiple Choice: (6 points each. No part credit.)

1-8	/48	11	/12
9	/20	12	/12
10	/12	Total	/104

1. A point has spherical coordinates $(\rho,\phi,\theta) = \left(6, \frac{\pi}{6}, \frac{\pi}{4}\right)$. Find its cylindrical coordinates.

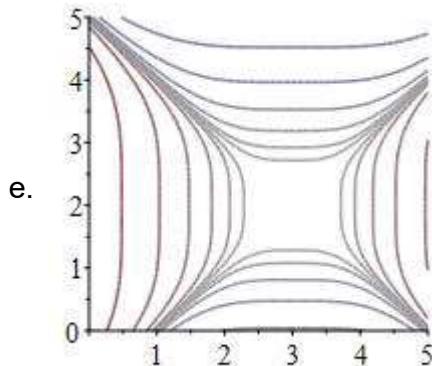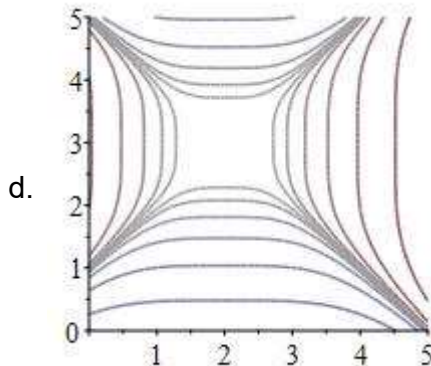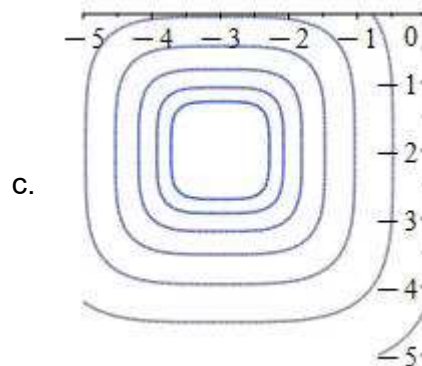   **a.** $(r,\theta,z) = \left(3, \frac{\pi}{6}, 3\sqrt{3}\right)$
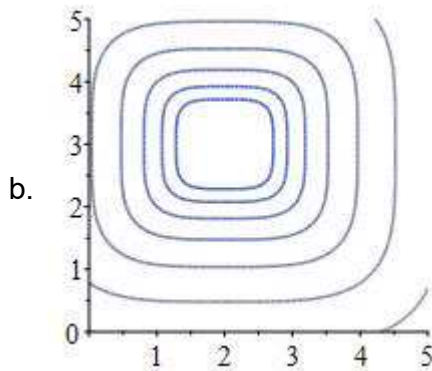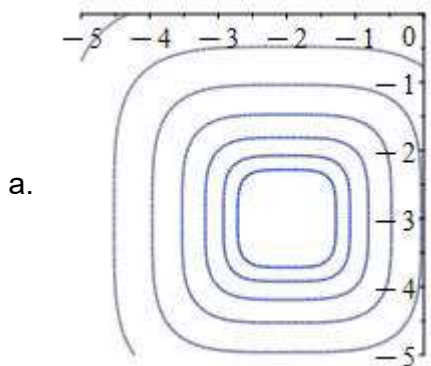
   b. $(r,\theta,z) = \left(3, \frac{\pi}{6}, 6\sqrt{3}\right)$

   c. $(r,\theta,z) = \left(3, \frac{\pi}{4}, 3\sqrt{3}\right)$

   d. $(r,\theta,z) = \left(6\sqrt{3}, \frac{\pi}{6}, 3\right)$

   e. $(r,\theta,z) = \left(3\sqrt{3}, \frac{\pi}{4}, 3\right)$

2. Which of the following is the contour plot of the function $f(x,y) = (x-2)^4 + (y-3)^4$?

a.

b.

c.

d.

e.

1

3. A hiker starts at the point $P = (4,2)$, travels along the vector $\vec{a} = \langle 2,-2 \rangle$, then along the vector $\vec{b} = \langle 1,3 \rangle$ and finally along the vector $\vec{c} = \langle -1,2 \rangle$. Along what vector should the hiker travel to get back to the starting point P?

    a. $\langle 2,3 \rangle$
    b. $\langle -6,-5 \rangle$
    c. $\langle 6,5 \rangle$
    d. $\langle -2,-3 \rangle$
    e. $\langle 2,-1 \rangle$

4. For what value of p is $\vec{u} = \langle p,5,3 \rangle$ perpendicular to $\vec{v} = \langle 2,1,p \rangle$?

    a. $p = 2$
    b. $p = 1$
    c. $p = 0$
    d. $p = -1$
    e. $p = -2$

5. Find the volume of the parallelepiped with edge vectors $\vec{a} = \langle 4,2,0 \rangle$, $\vec{b} = \langle 1,0,-3 \rangle$ and $\vec{c} = \langle 0,-1,2 \rangle$.

    a. 16
    b. 12
    c. 8
    d. -12
    e. -16

6. If $\hat{T}$ points Up and $\hat{B}$ points NorthEast, in what direction does $\hat{N}$ point?

    a. Down
    b. NorthWest
    c. SouthEast
    d. SouthWest
    e.

7. Which of the following is a plane perpendicular to the line $(x,y,z) = (1 + 3t, 3 + 2t, 4 - t)$?

    a. $3x + 2y - z = 7$
    b. $-3x + 2y + z = 2$
    c. $x + 3y + 4z = 5$
    d. $3x - 2y - z = 3$
    e. $x - 3y + 4z = 5$

8. Classify the quadratic surface: $-x^2 + 2x + y^2 + 4y - 2z^2 + 12z = 14$

    a. Hyperboloid of 1 sheet
    b. Hyperboloid of 2 sheets
    c. Hyperbolic Paraboloid opening up in the x-direction and down in the y-direction
    d. Hyperbolic Paraboloid opening up in the y-direction and down in the x-direction
    e. Cone

9. (20 pts) Consider the twisted cubic $\vec{r} = (6t, 3t^2, t^3)$. Compute each of the following.
Note: $4 + 4t^2 + t^4 = (2 + t^2)^2$

    a. (6 pts) Arc length between $(0,0,0)$ and $(6,3,1)$.

    b. (6 pts) Curvature $\kappa = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$.

       HINT: Factor out an 18^2.

    c. (4 pts) Tangential acceleration, a_T.
       HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

    d. (4 pts) Normal acceleration, a_N.
       HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

10. (12 pts) Write the vector, $\vec{a} = \langle 5,1,-3 \rangle$, as a sum of two vectors $\vec{p}$ and $\vec{q}$, where $\vec{p}$ is parallel to $\vec{b} = \langle 6,4,2 \rangle$ and $\vec{q}$ is perpendicular to $\vec{b}$.

11. (12 pts) Consider the helix $\vec{r}(\theta) = \langle 3\cos\theta, 3\sin\theta, 4\theta \rangle$ for $0 \le \theta \le 2\pi$.

a. Find its mass, if its linear density is $\delta(x,y,z) = z$.

b. Find the work done to push a bead along the helix if the force is $\vec{F} = \langle -2y, 2x, 0 \rangle$.

12. (12 pts) Consider the planes:

$$P_1 : \qquad x + 3y + 3z = 5$$
$$P_2 : \qquad x + y - z = 3$$

Determine if they are parallel or intersecting. If they intersect, find the line of intersection. You MUST explain why they are or are not parallel.