1-6	/54
7	/20
8	/15
9	/15
Total	/104

Name________________________________

MATH 221 Exam 3, Version A Spring 2024

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Multiple Choice: (9 points each. No part credit.)
Circle you answers here and bubble on the Scantron.
Show your work, in case I give some part credit.

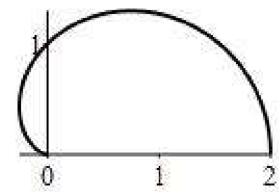
1. Compute $\int_0^2 \int_p^{2p} pq \, dq \, dp$.

- **a**. 4
- **b**. 6
- **c**. 8
- **d**. 12
- **e**. 24

2. Approximate the integral $\int_2^6 \int_1^5 (x^2 + y) \, dy \, dx$ by a Riemann sum using 4 squares and evaluating at the center of each square.

- **a**. 320
- **b**. $\frac{976}{3}$
- **c**. $\frac{488}{3}$
- **d**. 160
- **e**. 80

3. Find the area of the interior of the upper half of the cardioid $r = 1 + \cos\theta$.



a. $A = \pi$

b. $A = 2\pi$

c. $A = \dfrac{2}{3\pi}$

d. $A = \dfrac{3\pi}{4}$

e. $A = \dfrac{3\pi}{2}$

4. Find the average value of the function $f(x,y) = y$ on the interior of the upper half of the cardioid $r = 1 + \cos\theta$ as shown in problem 3.

a. $f_{\text{ave}} = \dfrac{3}{4}$

b. $f_{\text{ave}} = \dfrac{4}{3}$

c. $f_{\text{ave}} = \dfrac{4}{9\pi}$

d. $f_{\text{ave}} = \dfrac{9\pi}{16}$

e. $f_{\text{ave}} = \dfrac{16}{9\pi}$

5. Find the volume of the apple given in spherical coordinates by $\rho = 1 - \cos\phi$.

a. $\frac{4\pi}{5}$

b. $\frac{8\pi}{5}$

c. $\frac{4\pi}{3}$

d. $\frac{8\pi}{3}$

e. $2\pi^2$

6. Find the surface area of the parametric surface parametrized by $\vec{R}(u,v) = \langle u+v, u-v, 2u+2v \rangle$ for $0 \le u \le 2$ and $0 \le v \le 3$.

a. 36

b. 6

c. $2\sqrt{5}$

d. $6\sqrt{5}$

e. $12\sqrt{5}$

Work Out: (Points indicated. Part credit possible. Show all work.)

7. (20 points) Consider the region between the y-axis and the parabola $x = 9 - y^2$.

a. Find the mass of the region, if the surface density is $\delta = y^2$.

b. Find the center of mass of the region, if the surface density is $\delta = y^2$.

Write your answer as an ordered pair.

8. (15 points) Compute $\iiint_P \vec{\nabla} \cdot \vec{F}\, dV$ for $\vec{F} = \langle xz, yz, z^2 \rangle$ over the solid above the paraboloid $z = x^2 + y^2$ below the plane $z = 4$.

9. (15 points) Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = \langle -yz, xz, z^2 \rangle$ over the parabolic surface $z = x^2 + y^2$ below $z = 4$ oriented down and out. It may be parametrized by

$$\vec{R}(r,\theta) = \langle r\cos\theta, r\sin\theta, r^2 \rangle$$