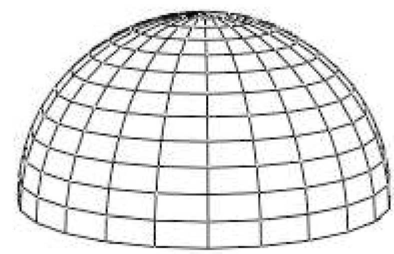
Name________________________ Section____________

MATH 221 Take Home Quiz 4 P. Yasskin

1. (20 points) Verify Stokes' Theorem $\iint\limits_H \vec\nabla \times \vec F \bullet d\vec S = \oint\limits_{\partial H} \vec F \bullet d\vec s$



for the vector field $\vec F = (y, -x, xz + yz)$

and the hemisphere $z = \sqrt{9 - x^2 - y^2}$ oriented up.

Use the following steps:

a. Parametrize the surface and compute the surface integral:

$\vec R(\varphi, \theta) =$

$\vec e_\varphi =$

$\vec e_\theta =$

$\vec N =$ (Check the orientation)

$\vec\nabla \times \vec F =$

$\vec\nabla \times \vec F\Big|_{\vec R(\varphi,\theta)} =$

$\iint\limits_H \vec\nabla \times \vec F \bullet d\vec S =$

1. continued. Recall: $\vec{F} = (y, -x, xz + yz)$ and $z = \sqrt{9 - x^2 - y^2}$.

b. Parametrize the boundary curve and compute the line integral:

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}\big|_{\vec{r}(\theta)} =$

$$\oint_{\partial H} \vec{F} \cdot d\vec{s} =$$

2. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \bullet \vec{F}\,dV = \iint\limits_{\partial V} \vec{F} \bullet d\vec{S}$

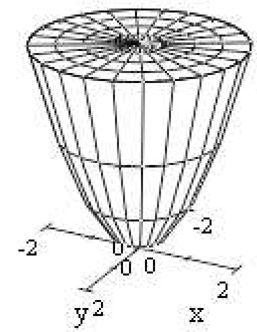
for the vector field $\vec{F} = (xz, yz, z^2)$ and

the volume above the paraboloid P: $z = x^2 + y^2$ for $z \le 4$

and below the disk D: $x^2 + y^2 \le 4$ with $z = 4$.

Be sure to check and explain the orientations.

Use the following steps.



a. Compute the divergence $\vec{\nabla} \bullet \vec{F}$ and the volume integral $\iiint\limits_V \vec{\nabla} \bullet \vec{F}\,dV$.

$\vec{\nabla} \bullet \vec{F} =$

$\iiint\limits_V \vec{\nabla} \bullet \vec{F}\,dV =$

b. Parametrize the disk, D, and compute the surface integral:

$\vec{R}(r,\theta) =$

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$ (Check the orientation)

$\vec{F}\big|_{\vec{R}(r,\theta)} =$

$\vec{F} \bullet \vec{N} =$

$\iint\limits_D \vec{F} \bullet d\vec{S} =$

c. The paraboloid, P, may be parametrize as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.
Compute the surface integral:

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$ (Check the orientation)

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

$\vec{F} \cdot \vec{N} =$

$\iint\limits_P \vec{F} \cdot d\vec{S} =$

d. Combine $\iint\limits_D \vec{F} \cdot d\vec{S}$ and $\iint\limits_P \vec{F} \cdot d\vec{S}$ to get $\iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$.

$\iint\limits_{\partial V} \vec{F} \cdot d\vec{S} =$