Name (print): ______________________ Instructor: Yasskin

Signature: ______________________ Section: 537,538,539

1-14	/56
15	/10
16	/10
17	/30
Total	/106

MATH 151, Fall 2013

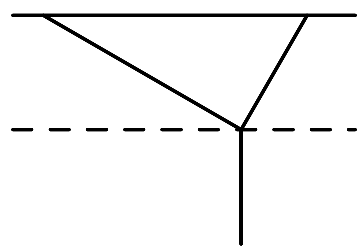
FINAL EXAMINATION

Part 1 – Multiple Choice (**14 questions**, **4 points each**, **No Calculators**)

Write your name and section number on the ScanTron form.

Mark your responses on the ScanTron form and on the exam itself

1. A ball whose weight is $F = 50$ Newtons hangs from two wires, one at angle $30°$ above horizontal on the left, and the other at angle $60°$ above horizontal on the right. Let $\vec{T}_1$ be the tension in the first wire, and $\vec{T}_2$ be the tension in the second wire. Which set of equations can be used to solve for $|\vec{T}_1|$ and $|\vec{T}_2|$?



a. $-\frac{\sqrt{3}}{2}|\vec{T}_1| + \frac{1}{2}|\vec{T}_2| = 0 \qquad \frac{1}{2}|\vec{T}_1| + \frac{\sqrt{3}}{2}|\vec{T}_2| = 50$

b. $-\frac{1}{2}|\vec{T}_1| + \frac{\sqrt{3}}{2}|\vec{T}_2| = 0 \qquad \frac{\sqrt{3}}{2}|\vec{T}_1| + \frac{1}{2}|\vec{T}_2| = 50$

c. $\frac{\sqrt{3}}{2}|\vec{T}_1| + \frac{1}{2}|\vec{T}_2| = 0 \qquad -\frac{1}{2}|\vec{T}_1| + \frac{\sqrt{3}}{2}|\vec{T}_2| = 50$

d. $\frac{1}{2}|\vec{T}_1| + \frac{\sqrt{3}}{2}|\vec{T}_2| = 0 \qquad -\frac{\sqrt{3}}{2}|\vec{T}_1| + \frac{1}{2}|\vec{T}_2| = -50$

e. $-\frac{1}{2}|\vec{T}_1| + \frac{\sqrt{3}}{2}|\vec{T}_2| = 0 \qquad \frac{\sqrt{3}}{2}|\vec{T}_1| + \frac{1}{2}|\vec{T}_2| = -50$

2. Find the cosine of the angle between the vectors $\vec{A} = \langle 3,4 \rangle$ and $\vec{B} = \langle 12,-5 \rangle$

a. $\cos(\theta) = \frac{56}{\sqrt{65}}$

b. $\cos(\theta) = \frac{4}{\sqrt{65}}$

c. $\cos(\theta) = \frac{16}{65}$

d. $\cos(\theta) = \frac{4}{65}$

e. $\cos(\theta) = \frac{56}{65}$

3. Evaluate $\lim_{x\to\infty} \frac{\sqrt{x^4 - 4x^3 - 5} - \sqrt{x^4 + 4x^3 + 5}}{x}$

a. $-\infty$
b. -8
c. -4
d. -2
e. 0

4. Which interval contains the unique real solution of the equation $x^3 + 2x = 4$

a. $(-2,-1)$
b. $(-1,0)$
c. $(0,1)$
d. $(1,2)$

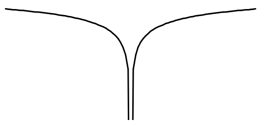
5. Find the 2017-th derivative of $f(x) = x + \ln x$

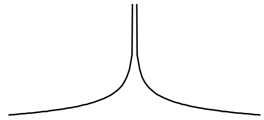
a. $2014!\,x^{-2015}$
b. $-2016!\,x^{-2017}$
c. $2015!\,x^{-2016}$
d. $2016!\,x^{-2017}$
e. $-2015!\,x^{-2016}$

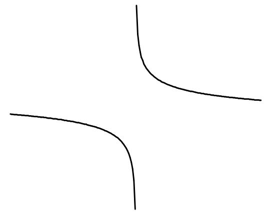
6. Find the line tangent to the curve $y^3 = x^2 - xy$ at $(x,y) = (-2,2)$. Its y-intercept is

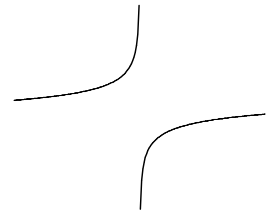
a. $\frac{4}{5}$
b. $\frac{3}{5}$
c. $-\frac{3}{5}$
d. $-\frac{4}{5}$
e. -2

7. Near $x = 2$, the graph of $f(x) = (x-2)^{-1/5}$ has the shape:

a. 

b. 

c. 

d. 

e None of these

8. $\lim\limits_{x \to 0} \dfrac{x \sin x}{\cos x - 1} =$

a. -2
b. -1
c. 0
d. 1
e. 2

9. Water is poured onto a tabletop at $\dfrac{dV}{dt} = 3$ cm$^3/\mathrm{sec}$ in such a way that it forms a circular puddle of radius r and constant height of $h = 0.2$ cm. Find the rate that the radius is increasing when it is $r = 5$ cm.
HINT: The puddle is actually a cylinder. What is its volume?

a. $\dfrac{1}{2\pi}$
b. $\dfrac{2\pi}{3}$
c. π
d. $\dfrac{3}{2\pi}$
e. 2π

10. Find the points of inflection of the function $f(x) = \dfrac{1}{20}x^5 - \dfrac{1}{3}x^4$.

a. $x = 0$ and $x = 4$ only
b. $x = 4$ only
c. $x = 0$ only
d. $x = -2$ and $x = 2$ only
e. $x = -4$ and $x = 0$ only

11. If the initial position and velocity of a car are $x(0) = 3$ and $v(0) = 2$ and its accereration is $a(t) = \sin t + 6t$ find its position at $t = 1$.

a. $7 + \sin 1$
b. $3 + \cos 1$
c. $6 + \sin 1$
d. $7 - \sin 1$
e. $3 - \cos 1$

12. If $A(z)$ is the area under the parabola $y = 2 + x^2$ between $x = 1$ and $x = z$ find $A'(3)$.

a. 6
b. 8
c. 9
d. 11
e. 13

13. Find the area under the parabola $y = 2 + x^2$ between $x = 1$ and $x = 4$

a. 21
b. 24
c. 27
d. 30
e. 36

14. Approximate the area under the parabola $y = 1 + x^2$ between $x = 1$ and $x = 7$ using a Right Riemann Sum with 3 equal intervals and right endpoints.

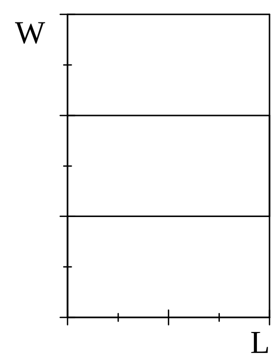
a. 38
b. 76
c. 86
d. 120
e. 172

Part 2 – Work Out Problems (3 questions. Points indicated. No Calculators)

Solve each problem in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

15. (10 points) Let $f(x) = xe^x$ and $g(x) = f^{-1}(x)$ the inverse function of $f(x)$. Find $g'(e)$.

16. (10 points) A rectangular field is surrounded by a fence and divided into 3 pens by 2 additional fences parallel to one side. If the total area is 50 m^2 find the minimum total length of fence.



17. (30 points) Analyze the graph of the function $f(x) = xe^{-2x^2}$. Find each of the following. If none or nowhere or undefined, say so.

a. y-intercepts:

b. x-intercepts:

c. horizontal asymptote as $x \to +\infty$:

d. horizontal asymptote as $x \to -\infty$:

e. vertical asymptotes:

f. first derivative:

$f'(x) =$

g. critical points:

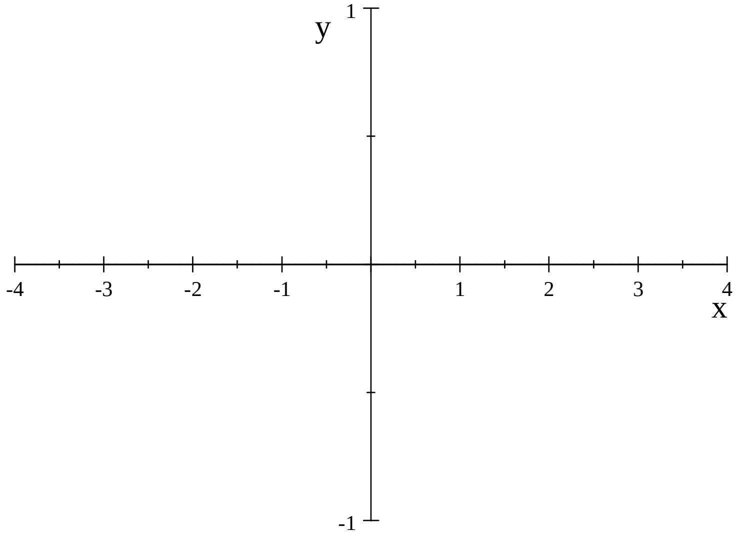
h. intervals where increasing:

i. intervals where decreasing:

j. local maxima $\left(x \text{ and } y \text{ coordinates}\right)$:

k. local minima $\left(x \text{ and } y \text{ coordinates}\right)$:

l. Roughly graph the function:



m. second derivative:

$f''(x) =$

n. inflection points $\left(x \text{ coordinate only}\right)$:

o. intervals where concave up:

p. intervals where concave down: