MATH 151, FALL 2013
COMMON EXAM I - VERSION A

LAST NAME: Key ______________________ FIRST NAME: ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: ____________

UIN: ______________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-22), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		6
17		8
18		5
19		10
20		10
21		8
22		8
		100

PART I: Multiple Choice. 3 points each

1. If $\mathbf{a} = \langle 1,1\rangle$, $\mathbf{b} = \langle 2,1\rangle$ and $\mathbf{c} = \langle 4,-3\rangle$, what value of t satisfies $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$, where s and t are scalars?

   (a) $t = -7$
   (b) $t = 7$
   (c) $t = -10$
   (d) $t = 1$
   (e) $t = 2$

$\langle 4,-3\rangle = s\langle 1,1\rangle + t\langle 2,1\rangle$

$\langle 4,-3\rangle = \langle s+2t, s+t\rangle$

$s+2t=4 \rightarrow s = 4-2t$

$s+t=-3 \rightarrow 4-2t+t=-3$

$\rightarrow -t=-7 \quad \boxed{t=7}$

2. Find $\lim_{x\to 3^-} \frac{|x-3|}{x^2-2x-3}$

   (a) $-\frac{1}{4}$
   (b) 0
   (c) ∞
   (d) $\frac{1}{4}$
   (e) The limit does not exist

for $x<3$, $|x-3| = -(x-3)$

$$\lim_{x\to 3^-} \frac{|x-3|}{x^2-2x-3} = \lim_{x\to 3^-} \frac{-\cancel{(x-3)}}{\cancel{(x-3)}(x+1)}$$

$$= -\frac{1}{4}$$

3. Find the vector projection of $\langle -3,1\rangle$ onto $\langle 2,5\rangle$.

   (a) $\left\langle -\frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}}\right\rangle$
   (b) $\left\langle \frac{3}{10}, -\frac{1}{10}\right\rangle$
   (c) $\left\langle \frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right\rangle$
   (d) $\left\langle -\frac{2}{29}, -\frac{5}{29}\right\rangle$
   (e) $\left\langle \frac{11}{29}, \frac{55}{29}\right\rangle$

let $\vec{b} = \langle -3,1\rangle$, $\vec{a} = \langle 2,5\rangle$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$$

$$= \frac{\langle 2,5\rangle\cdot\langle -3,1\rangle}{|\langle 2,5\rangle|^2}\langle 2,5\rangle$$

$$= \frac{-6+5}{(\sqrt{29})^2}\langle 2,5\rangle = -\frac{1}{29}\langle 2,5\rangle$$

4. Find $\lim_{x\to -1^-} \frac{x-2}{x+1}$

   (a) ∞
   (b) 0
   (c) 1
   (d) $-\infty$
   (e) The limit does not exist

$\lim_{x\to -1^-} \frac{x-2}{x+1} = \infty$ since $\frac{x-2}{x+1} > 0$ for $x \to -1^-$

5. Given the points $A(0,1)$, $B(2,0)$ and $C(3,-4)$, find the angle, α, located at the vertex A. That is, $\angle BAC$.

(a) $\alpha = \arccos\left(\frac{11}{\sqrt{170}}\right)$

(b) $\alpha = \arccos\left(-\frac{11}{\sqrt{170}}\right)$

(c) $\alpha = \arccos\left(\frac{1}{\sqrt{170}}\right)$

(d) $\alpha = \arccos\left(\frac{-6}{\sqrt{85}}\right)$

(e) $\alpha = \arccos\left(\frac{2}{\sqrt{85}}\right)$

$\vec{AB} = \langle 2, -1 \rangle$

$\vec{AC} = \langle 3, -5 \rangle$

$\cos\alpha = \frac{\langle 2,-1\rangle \cdot \langle 3,-5\rangle}{|\langle 2,-1\rangle|\,|\langle 3,-5\rangle|} = \frac{11}{\sqrt{5}\sqrt{34}}$

$\alpha = \arccos\left(\frac{11}{\sqrt{170}}\right)$

6. Find $\lim_{t\to 4} \mathbf{r(t)}$ where $\mathbf{r(t)} = \left\langle 2t+1, \frac{\sqrt{t+5}-3}{t-4} \right\rangle$. $= \langle 9, \frac{1}{6} \rangle$

(a) $\langle 9, 0 \rangle$

(b) $\langle 9, 1 \rangle$

(c) $\left\langle 9, \frac{1}{6} \right\rangle$

(d) $\left\langle 9, -\frac{1}{6} \right\rangle$

(e) $\left\langle 9, -\frac{1}{2} \right\rangle$

$\lim_{t\to 4} (2t+1) = 9$

$\lim_{t\to 4} \frac{\sqrt{t+5}-3}{t-4} \cdot \frac{\sqrt{t+5}+3}{\sqrt{t+5}+3}$

$\lim_{t\to 4} \frac{t+5-9}{(t-4)(\sqrt{t+5}+3)} = \lim_{t\to 4} \frac{\cancel{t-4}}{(\cancel{t-4})(\sqrt{t+5}+3)} = \frac{1}{6}$

7. Find the horizontal and vertical asymptotes for $f(x) = \frac{(2-x)(3x+1)}{x^2-4}$.

(a) $x = -3$, $y = -2$

(b) $y = -3$, $x = 2$, $x = -2$

(c) $x = -3$, $y = 2$, $y = -2$

(d) $y = -3$, $x = -2$

(e) $y = 3$, $x = -2$

$f(x) = \frac{(2-x)(3x+1)}{(x+2)(x-2)}$

Horizontal asymptote: $y = -3$

Vertical asymptote: $x = -2$

8. Find a unit vector in the direction of $\mathbf{a} - \mathbf{b}$ where $\mathbf{a} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = -5\mathbf{j}$.

(a) $\frac{6}{\sqrt{45}}\mathbf{i} - \frac{3}{\sqrt{45}}\mathbf{j}$

(b) $\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$

(c) $\frac{1}{\sqrt{65}}\mathbf{i} - \frac{2}{\sqrt{65}}\mathbf{j}$

(d) $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$

(e) $\frac{8}{\sqrt{65}}\mathbf{i} - \frac{1}{\sqrt{65}}\mathbf{j}$

$\vec{a} = \langle 1, -3 \rangle$

$\vec{b} = \langle 0, -5 \rangle$

$a - b = \langle 1, -3 \rangle - \langle 0, -5 \rangle$

$a - b = \langle 1, 2 \rangle$

$a - b = \vec{i} + 2\vec{j}$

$\vec{u} = \frac{i + 2j}{|i+2j|}$

$\vec{u} = \frac{\vec{i} + 2\vec{j}}{\sqrt{5}} = \frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}j$

9. Which interval contains a solution to the equation $x^3 + x = 3$?

(a) $[-1, 0]$
(b) $[0, 2]$
(c) $[0, 1]$
(d) $[-2, -1]$
(e) $[2, 4]$

let $f(x) = x^3 + x$, a continuous function

$N = 3$

since $f(0) = 0 < 3$

and $f(2) = 10 > 3$, a solution to $f(x) = 3$ exists on $[0,2]$

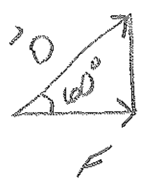
10. Consider $f(x) = \begin{cases} x^2 + 5x + 1 & \text{if } x < -1 \\ 3 & \text{if } x = -1 \\ 2x - 1 & \text{if } x > -1 \end{cases}$. Why is $f(x)$ not continuous at $x = -1$?

(a) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1} f(x) \neq f(-1)$.
(b) $f(x)$ is not continuous at $x = -1$ because $f(-1)$ does not exist.
(c) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1} f(x)$ does not exist.
(d) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1^-} f(x)$ does not exist.
(e) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1^+} f(x)$ does not exist.

$\lim_{x \to -1} f(x) = -3$, but $f(-1) = 3$

11. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of 60° above the horizontal. Find the work done on the box.

(a) $50\sqrt{3}$ foot pounds
(b) $50\sqrt{2}$ foot pounds
(c) 100 foot pounds
(d) 10 foot pounds
(e) 50 foot pounds



$W = |F||D|\cos 60°$

$= (20 \text{ lbs})(5 \text{ feet})(\frac{1}{2})$

$= 50 \text{ ft-lbs}$

12. Find $\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$. $= \lim_{x \to 2} \frac{(x^2+4)(x^2-4)}{x-2}$

(a) 0
(b) ∞
(c) 4
(d) 32
(e) 1

$= \lim_{x \to 2} \frac{(x^2+4)(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= \boxed{32}$

13. Find the value of x so that the vectors $\langle 4, x+1 \rangle$ and $\langle x, 3 \rangle$ are perpendicular.

(a) $x = 0$

(b) $x = -\frac{7}{3}$

(c) $x = -\frac{3}{7}$

(d) $x = -\frac{1}{7}$

(e) $x = \frac{1}{7}$

$\langle 4, x+1 \rangle \cdot \langle x, 3 \rangle = 0$

$4x + 3x + 3 = 0$

$7x = -3$

$x = \frac{-3}{7}$

14. Find the average rate of change of $f(t) = \sqrt{2t+3}$ from $t = 1$ to $t = 3$.

(a) $3 - \sqrt{5}$

(b) $\frac{3+\sqrt{5}}{2}$

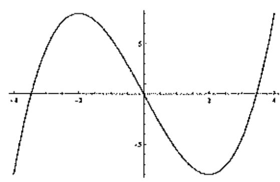
(c) $\frac{\sqrt{5}-3}{2}$

(d) $3 + \sqrt{5}$

(e) $\frac{3-\sqrt{5}}{2}$

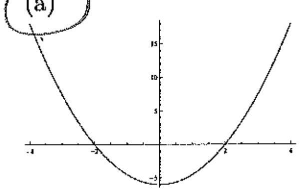
$APOC = \frac{f(3) - f(1)}{3-1} = \frac{3-\sqrt{5}}{2}$

15. Consider The graph of $f(x)$ given:

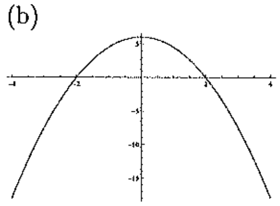


Which of the following is the graph of its derivative, $f'(x)$?

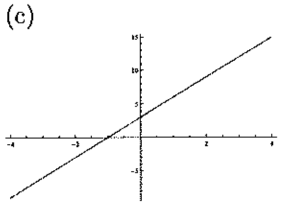
(a)



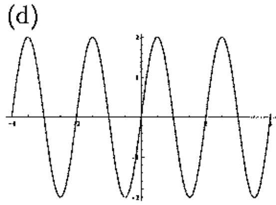
(b)



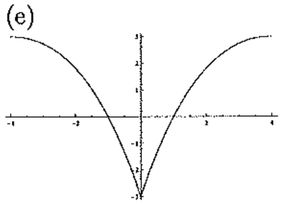
(c)



(d)



(e)



PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Consider the line $x = 8 - 2t$, $y = 14 + 7t$.

(i) (2 pts) Find a vector parallel to the line.

$v = \langle -2, 7 \rangle$ or any scalar multiple of this vector

(ii) (2 pts) Find a vector perpendicular to the line.

$v^{\perp} = \langle -7, -2 \rangle$ or any scalar multiple of this vector

(iii) (2 pts) Find the x and y intercepts of the line.

x-int: $y = 0 \to 0 = 14 + 7t$

$t = -2 \to \boxed{x = 12}$

y-int: $x = 0 \to 0 = 8 - 2t \to t = 4 \to \boxed{y = 42}$

17. (8 pts) Find $\lim\limits_{x\to\infty} \left(\sqrt{x^2+6x-1} - x\right)$.

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+6x-1} - x\right)\left(\sqrt{x^2+6x-1} + x\right)}{\sqrt{x^2+6x-1} + x}$$

$$= \lim_{x\to\infty} \frac{x^2+6x-1-x^2}{\sqrt{x^2+6x-1}+x}$$

$$= \lim_{x\to\infty} \frac{6x-1}{\sqrt{x^2+6x-1}+x}$$

$$\to \lim_{x\to\infty} \frac{\frac{6x-1}{x}}{\frac{\sqrt{x^2+6x-1}+x}{x}}$$

$$\lim_{x\to\infty} \frac{6 - \cancel{\frac{1}{x}}^{\,0}}{\sqrt{1 + \cancel{\frac{6}{x}}^{\,0} - \cancel{\frac{1}{x^2}}^{\,0}} + 1} \quad \text{(2pts)} \quad = \frac{6}{2}$$

$$= \boxed{3}$$

18. (5 pts) If $f(2) = 3$ and $f'(2) = -7$, find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

$m = f'(2) = -7$

point $(2, f(2)) = (2, 3)$

$y - 3 = -7(x-2)$ or $y = -7x + 17$

19. (10 pts) For $f(x) = \dfrac{1}{2x+1}$, find $f'(x)$ using the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2x+2h+1} - \frac{1}{2x+1}}{h}$$

$$= \lim_{h \to 0} \frac{2x+1-(2x+2h+1)}{h(2x+2h+1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(2x+2h+1)(2x-1)}$$

$$= \frac{-2}{(2x+1)^2}$$

20. (10 pts) Consider $f(x) = \begin{cases} cx+2 & \text{if } x > 5 \\ \frac{1}{2} & \text{if } x = 5 \\ cx^2 - 4 & \text{if } x < 5 \end{cases}$

(i) Find $\lim_{x \to 5^+} f(x)$ in terms of c.

$\lim_{x \to 5^+} f(x) = 5c + 2$

(ii) Find $\lim_{x \to 5^-} f(x)$ in terms of c.

$\lim_{x \to 5^-} f(x) = 25c - 4$

(iii) For what value of c does $\lim_{x \to 5} f(x)$ exist?

$5c + 2 = 25c - 4$

$6 = 20c$

$c = \frac{6}{20} = \frac{3}{10}$

(iv) For the value of c found above, what is $\lim_{x \to 5} f(x)$?

$$f(x) = \begin{cases} \frac{3}{10}x + 2 & x > 5 \\ \frac{1}{2} & x = 5 \\ \frac{3}{10}x^2 - 4 & x < 5 \end{cases}$$

$\lim_{x \to 5} f(x) = \frac{15}{10} + 2$

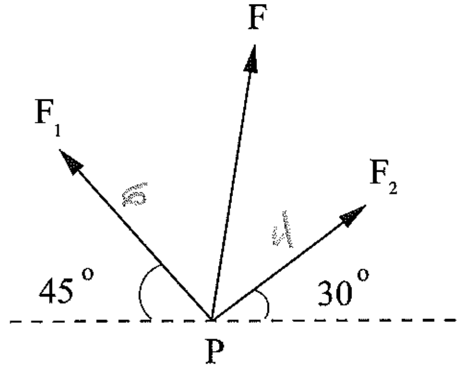
$= \frac{7}{2}$

(v) For the value of c above, is $f(x)$ continuous at $x = 5$? Support your answer.

$\lim_{x \to 5} f(x) = \frac{7}{2}$, $f(5) = \frac{1}{2}$

since $\lim_{x \to 5} f(x) \neq f(5)$, $f(x)$ is not continuous at $x = 5$.

21. (8 pts) Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ with magnitudes 6 lbs and 4 lbs, respectively, act on an object at a point P as shown.



(i) Find the vector, $\mathbf{F_1}$. Evaluate trig functions.

$F_1 = \langle -6\frac{\sqrt{2}}{2}, 6\frac{\sqrt{2}}{2} \rangle$

$\boxed{F_1 = \langle -3\sqrt{2}, 3\sqrt{2} \rangle}$

(ii) Find the vector, $\mathbf{F_2}$. Evaluate trig functions.

$F_2 = \langle 4 \cdot \frac{\sqrt{3}}{2}, 4 \cdot \frac{1}{2} \rangle$

$\boxed{F_2 = \langle 2\sqrt{3}, 2 \rangle}$

(iii) Find the resultant force, $\mathbf{F}$, acting on the object.

$F = F_1 + F_2$

$\boxed{F = \langle -3\sqrt{2} + 2\sqrt{3}, 3\sqrt{2} + 2 \rangle}$

22. Consider the curve $x = 3 + \cos t$, $y = -1 + \sin t$.

(i) (4 pts) Eliminate the parameter to find a Cartesian equation.

$x - 3 = \cos t$

$y + 1 = \sin t$

$(x-3)^2 + (y+1)^2 = 1$

(ii) (4 pts) Sketch the curve on the grid below.