MATH 151, FALL 2013
COMMON EXAM II - VERSION B

LAST NAME: ____________________ FIRST NAME: ____________________

INSTRUCTOR: ____________________

SECTION NUMBER: __________

UIN: ____________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-21), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		10
17		6
18		12
19		12
20		8
21		7
		100

PART I: Multiple Choice. 3 points each

1. Find $\lim_{x\to\infty} \dfrac{2^x + 2^{-4x}}{7(2^x) - 2^{-4x}}$.

   (a) 0
   (b) $\dfrac{1}{7}$
   (c) ∞
   (d) $\dfrac{2}{7}$
   (e) -1

2. Find $h''(1)$ if $h(x) = e^{-x^2}$.

   (a) $\dfrac{4}{e}$
   (b) $-\dfrac{2}{e}$
   (c) $\dfrac{2}{e}$
   (d) $\dfrac{1}{e}$
   (e) $-\dfrac{4}{e}$

3. Consider the parametric curve $x(t) = t^4 + 1$ and $y(t) = \cos\left(\dfrac{\pi}{2}t\right)$. Find the slope of the tangent line at the point $(2, 0)$.

   (a) $-\dfrac{1}{4}$
   (b) $-\dfrac{\pi}{8}$
   (c) $-\dfrac{8}{\pi}$
   (d) 0
   (e) $-\dfrac{\pi}{4}$

4. The function $f(x) = 4x + \sin x$ is a one-to-one function. If g is the inverse of f, what is $g'(4\pi)$?

   (a) $-\dfrac{1}{5}$
   (b) -1
   (c) $\dfrac{1}{5}$
   (d) $\dfrac{1}{3}$
   (e) $\dfrac{1}{4}$

5. Find the quadratic approximation for $f(x) = e^{4x}$ at $x = 0$.

(a) $Q(x) = 1 + 4x + 16x^2$

(b) $Q(x) = 1 + x + \frac{1}{2}x^2$

(c) $Q(x) = 1 + x + x^2$

(d) $Q(x) = 1 + 4x + 2x^2$

(e) $Q(x) = 1 + 4x + 8x^2$

6. Use differentials to approximate $\sqrt[3]{8.2}$.

(a) $\sqrt[3]{8.2} \approx \frac{61}{60}$

(b) $\sqrt[3]{8.2} \approx \frac{121}{60}$

(c) $\sqrt[3]{8.2} \approx \frac{119}{60}$

(d) $\sqrt[3]{8.2} \approx \frac{22}{5}$

(e) $\sqrt[3]{8.2} \approx \frac{11}{5}$

7. At what point on the curve $x = 2t^2 - 4t$, $y = t^2 - t$ is the tangent line horizontal?

(a) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$

(b) $\left(\frac{1}{2}, -\frac{1}{4}\right)$

(c) $(0, 0)$

(d) $(-1, 0)$

(e) $(-2, 0)$

8. Find the slope of the tangent line to the curve $x^3 + y^3 = x^2 + 5y$ at the point $(2, 1)$.

(a) $m = 2$

(b) $m = -1$

(c) $m = \frac{11}{5}$

(d) $m = 4$

(e) $m = \frac{1}{4}$

9. Find the equation of the tangent line to the curve $g(x) = \dfrac{x}{2x+1}$ at $x = 4$.

(a) $y - \dfrac{4}{9} = \dfrac{1}{2}(x-4)$

(b) $y = \dfrac{1}{81}(x-4)$

(c) $y - \dfrac{4}{9} = \dfrac{1}{81}(x-4)$

(d) $y - \dfrac{4}{3} = \dfrac{1}{81}(x-4)$

(e) $y - \dfrac{4}{9} = \dfrac{1}{9}(x-4)$

10. $\lim\limits_{x \to 2^-} \left(\dfrac{1}{4}\right)^{\dfrac{x}{x-2}}$

(a) 0

(b) ∞

(c) $\dfrac{1}{4}$

(d) 1

(e) $-\infty$

11. If $g(x) = xf(x^3)$, $f(2) = 4$, $f(8) = 3$, $f'(8) = -1$, and $f'(2) = -2$, what is $g'(2)$?

(a) -24

(b) -12

(c) -16

(d) -44

(e) -21

12. Find the unit tangent vector to the curve $\mathbf{r}(\mathbf{t}) = \langle \sin(3t), \cos(3t) \rangle$ at $t = \dfrac{\pi}{9}$.

(a) $\left\langle -\dfrac{1}{2}, \dfrac{\sqrt{3}}{2} \right\rangle$

(b) $\left\langle \dfrac{1}{2}, -\dfrac{\sqrt{3}}{2} \right\rangle$

(c) $\left\langle \dfrac{\sqrt{3}}{2}, -\dfrac{1}{2} \right\rangle$

(d) $\left\langle -\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right\rangle$

(e) $\left\langle -\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2} \right\rangle$

13. Find $\lim_{t\to 0} \dfrac{\sin^2(6t)}{4t^2}$.

(a) $\dfrac{3}{2}$

(b) ∞

(c) $\dfrac{1}{9}$

(d) 0

(e) 9

14. Find the value(s) of x where the tangent line to the graph of $f(x) = x\sqrt{x}$ is parallel to the line $2x - 4y = 18$.

(a) $x = \dfrac{1}{9}$

(b) $x = \pm 9$

(c) $x = \dfrac{16}{3}$

(d) $x = 9$

(e) $x = \pm\dfrac{1}{9}$

15. An object is moving according to the equation of motion $s(t) = \cos t + \dfrac{1}{4}t^2$. Find the time(s) when the acceleration is zero for $0 \leq t \leq 2\pi$.

(a) $t = \dfrac{\pi}{3}$, $t = \dfrac{5\pi}{3}$

(b) $t = \dfrac{\pi}{6}$, $t = \dfrac{11\pi}{6}$

(c) $t = \dfrac{5\pi}{6}$, $t = \dfrac{7\pi}{6}$

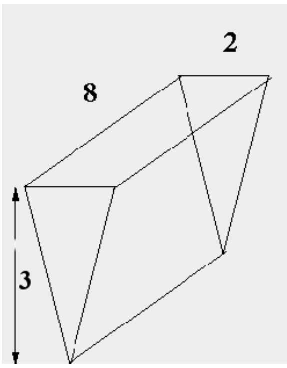
(d) $t = \dfrac{\pi}{4}$, $t = \dfrac{7\pi}{4}$

(e) $t = \dfrac{2\pi}{3}$, $t = \dfrac{4\pi}{3}$

PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (10 pts) A trough is 8 feet long. The ends of the trough are isosceles triangles with height 3 feet and width 2 feet across the top. If water is being poured into the trough at a rate of 1 cubic foot per minute, how fast is the water level rising when the depth of the water is 2 feet?



17. (6 pts) Find $\dfrac{dy}{dx}$ if $y = e^{2xy}$.

18. Find the derivative of the following functions. Do not simplify.

a.) (4 pts) $f(x) = \left((x^3+3)^7 + x\right)^6$

b.) (4 pts) $g(x) = \tan\left(\sqrt{3x^2+2x}\right)$

c.) (4 pts) $h(x) = \sin^4\left(a^3 + \dfrac{1}{x^3}\right)$

19. Given that $h(5) = 4$, $h'(5) = -3$, $g(5) = 7$ and $g'(5) = 3$, find $f'(5)$ for each of the following, if possible. If it is not possible, state what additional information is required.

a.) (3 pts) $f(x) = g(x)h(x)$

b.) (3 pts) $f(x) = \dfrac{g(x)}{h(x)}$

c.) (3 pts) $f(x) = [h(x)]^3$

d.) (3 pts) $f(x) = g(h(x))$

20. Consider the vector equation $\mathbf{r(t)} = \left\langle \sin\left(\frac{1}{t}\right), \sqrt{9-t^2} \right\rangle$.

a.) (2 pts) What is the domain of $\mathbf{r(t)}$? Use interval notation.

b.) (6 pts) Find $\mathbf{r'(t)}$ and the domain of $\mathbf{r'(t)}$. Use interval notation.

21. Consider $f(x) = \begin{cases} ax^2 + x + 3 & \text{if } x \leq -1 \\ bx - 2 & \text{if } x > -1 \end{cases}$.

a.) (4 pts) Find the value of a and b that make $f(x)$ differentiable everywhere.

b.) (3 pts) For the value of a and b found above, find $f'(x)$.