

**MATH 151 Honors, FALL SEMESTER 2011
FINAL EXAMINATION**

Name (print): _____ Instructor's name: Yasskin

Signature: _____ Section No: _____

Part 1 – Multiple Choice (15 questions, 4 points each, No Calculators)

Write your name and section number on the ScanTron form.
Mark your responses on the ScanTron form and on the exam itself

1. Evaluate $\lim_{x \rightarrow -2} \frac{x^3 - x^2 - 6x}{x^2 - 4}$

- a. 3
- b. ∞
- c. 0
- d. $-\frac{5}{2}$
- e. 6

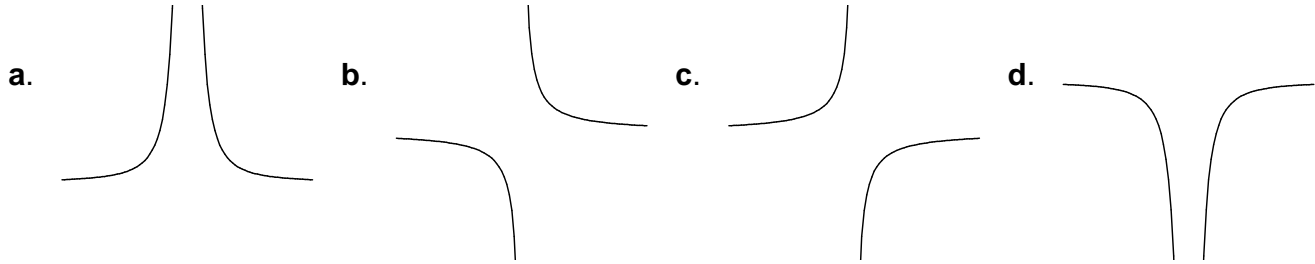
2. The limit $\lim_{h \rightarrow 0} \frac{4(2+h)^3 - 32}{h}$ can be interpreted as which of the following?

- a. $f'(32)$ where $f(x) = 4x^3$
- b. $f'(2)$ where $f(x) = 4x^3$
- c. $f'(4)$ where $f(x) = x^3$
- d. $f'(2)$ where $f(x) = 12x^2$
- e. $f'(2)$ where $f(x) = x^4$

3. Find the line tangent to $y = \sinh x$ at $x = 1$. Its y -intercept is

- a. 0
- b. $\frac{e}{2}$
- c. e
- d. $-\frac{1}{e}$
- e. $-\frac{1}{2e}$

4. The function $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$ has a vertical asymptote at $x = 2$. Near $x = 2$, the graph has the shape:



5. Find the absolute minimum value of $f(x) = \sin\left(x - \frac{\pi}{6}\right) - \frac{x}{2}$ on the interval $[0, \pi]$.

NOTE: $\frac{\sqrt{3}}{2} \approx 0.806$ $\frac{\pi}{4} \approx 0.785$ $\frac{\pi}{2} \approx 1.570$

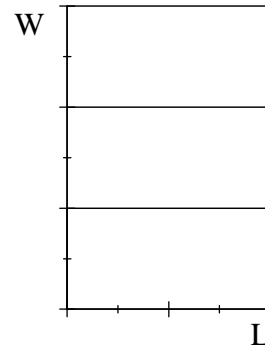
- a. $-\frac{1}{2}$
- b. $\frac{\sqrt{3}}{2} - \frac{\pi}{4}$
- c. $\frac{1}{2} - \frac{\pi}{2}$
- d. $-\frac{\sqrt{3}}{2} - \frac{\pi}{4}$
- e. $-\frac{1}{2} - \frac{\pi}{2}$

6. If \$1000 is invested at 2% annual interest compounded continuously, how long will it take for the principal to reach $\$1000e \approx \2718.28 ?

- a. 5 years
- b. 10 years
- c. 25 years
- d. 50 years
- e. 100 years

7. A rectangular field is surrounded by a fence and divided into 3 pens by 2 additional fences parallel to one side. If the total area is 72 m^2 find the minimum total length of fence.

- a. 48
- b. 50
- c. 52
- d. 60
- e. 66



8. Let $f(x)$ be a differentiable function, and suppose $f(1) = -5$ and $f'(x) \geq -7$ for all values of x . Use the Mean Value Theorem to determine how small $f(4)$ can possibly be.

- a. 26
- b. -26
- c. -20
- d. 35
- e. Not enough information.

9. When you prove $\lim_{x \rightarrow 4} (3x - 5) = 7$, and pick $\varepsilon > 0$, the largest possible δ is

- a. ε
- b. $\frac{\varepsilon}{3}$
- c. $\frac{\varepsilon}{2}$
- d. 2ε
- e. 3ε

10. When you use a left Riemann sum with 3 intervals of equal length to approximate $\ln 7 = \int_1^7 \frac{1}{x} dx$ you discover:
HINT: Draw a picture.

- a. $\ln 7 > \frac{46}{15}$
- b. $\ln 7 < \frac{46}{15}$
- c. $\ln 7 > \frac{11}{6}$
- d. $\ln 7 < \frac{11}{6}$
- e. $\ln 7 < \frac{23}{15}$

11. Evaluate $\int_{-\pi/4}^{\pi/4} \cos(3x) dx$

- a. $\sqrt{2}$
- b. $3\sqrt{2}$
- c. 0
- d. $\frac{\sqrt{2}}{3}$
- e. $-\sqrt{2}$

12. If $I = \int_0^2 2^{(x^2)} dx$, which of the following is FALSE?

- a. $\int_0^2 3 \cdot 2^{(x^2)} dx = 3I$
- b. $2 \leq I \leq 32$
- c. $\int_0^2 \sqrt{2^{(x^2)}} dx = \sqrt{I}$
- d. $\int_0^5 2^{(x^2)} dx - \int_2^5 2^{(x^2)} dx = I$
- e. $\int_2^0 2^{(x^2)} dx = -I$

13. Compute $\frac{d}{dx} \int_{x^3}^{x^2} \sin(t^2) dt$

- a. $\sin(x^4)2x - \sin(x^6)3x^2$
- b. $\cos(x^2)2x - \cos(x^3)3x^2$
- c. $\sin(x^4) - \sin(x^6)$
- d. $\cos(x^2) - \cos(x^3)$
- e. $-\cos(x^4)2x + \cos(x^6)3x$

14. Recall: $\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$.

Then $\cosh(x)\cosh(2x) - \sinh(x)\sinh(2x) =$

- a. $\sinh(3x)$
- b. $\sinh(x)$
- c. $\cosh(3x)$
- d. $\cosh(x)$
- e. $-\cosh(x)$

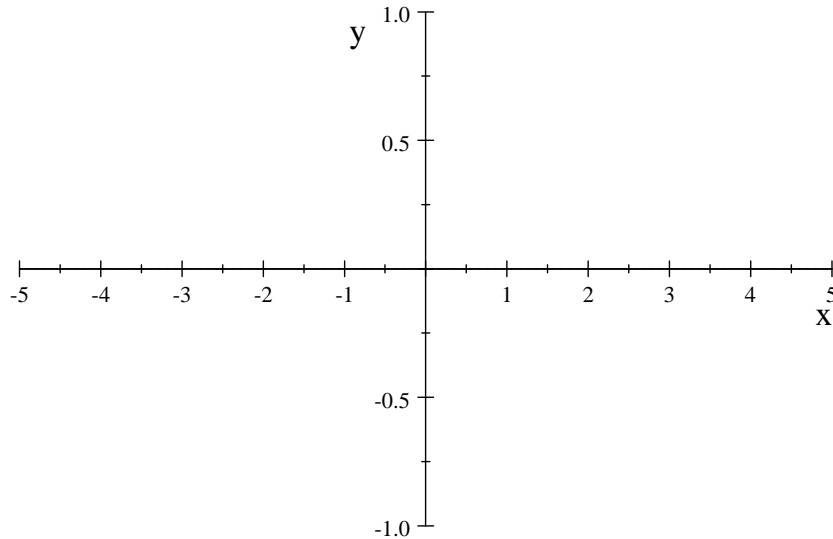
15. Find the intervals of concavity of the function $f(x) = 12(12 + x^2)^{-1}$.

- a. Concave up: $(2, \infty)$ Concave down: $(-\infty, -2) \cup (-2, 2)$
- b. Concave up: $(-\infty, -2) \cup (2, \infty)$ Concave down: $(-2, 2)$
- c. Concave up: $(-2, 2)$ Concave down: $(-\infty, -2) \cup (2, \infty)$
- d. Concave up: $(-\infty, \infty)$ Concave down: nowhere
- e. Concave up: nowhere Concave down: $(-\infty, \infty)$

Part 2 – Work Out Problems (5 questions. Points indicated. No Calculators)

Solve each problem in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (6 points) Graph the function $f(x) = 12(12 + x^2)^{-1}$. (See problem #15.)
Be sure to label all maxima, minima, inflection points and asymptotes.
Be careful with intervals of increase, decrease and concavity.

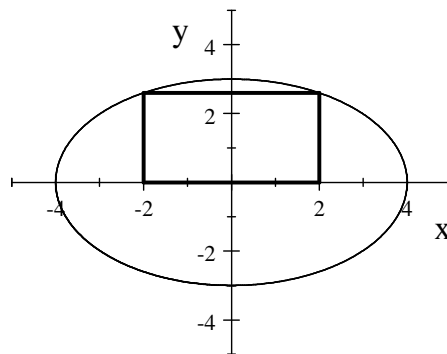


17. (8 points) Let $g(x)$ be the inverse function of $f(x) = \frac{1}{\ln x}$ for $x > 0$. Find $g(1)$ and $g'(1)$.

18. (10 points) The pressure and volume of the gas in an engine cylinder are related by $PV^{3/2} = C$ for some constant C . Currently, the pressure is $P = 15 \text{ lb/in}^2$ and the volume is $V = 4 \text{ in}^3$ but is increasing at the rate of $\frac{dV}{dt} = 2 \text{ in}^3/\text{min}$. Find C and the current value of $\frac{dP}{dt}$. Is the pressure currently increasing or decreasing?

19. (6 points) Find a parametric equation for the line perpendicular to the parametric curve $\vec{r}(t) = \langle t^2, t^3 \rangle$ at the point $(4, 8)$.

20. (12 points) A rectangle is inscribed in the upper half of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with its base on the x -axis. Find the maximum area of such a rectangle.



Question	Points/Max
1-15	/60
16	/6
17	/8
18	/10
19	/6
20	/12
Total	/102

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