Name (print): ______________________________ Instructor: Yasskin

Signature: ______________________________ Section: 201 202

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13	/10
14	/30
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16	/10
Total	/103

MATH 151 Honors, Fall 2013

FINAL EXAMINATION

Part 1 – Multiple Choice (**12 questions**, **4 points each**, **No Calculators**)

Write your name and section number on the ScanTron form.
Mark your responses on the ScanTron form and on the exam itself

1. Find the 2017-th derivative of $f(x) = x\ln x$

a. $2014!\,x^{-2015}$
b. $-2016!\,x^{-2017}$
c. $2015!\,x^{-2016}$
d. $2016!\,x^{-2017}$
e. $-2015!\,x^{-2016}$

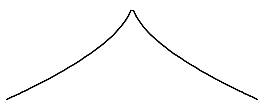
2. Evaluate $\lim\limits_{x\to\infty}\left(\sqrt{x^2-4x+5}-\sqrt{x^2+4x+5}\right)$

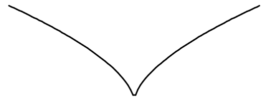
a. $-\infty$
b. -4
c. 0
d. 2
e. 4

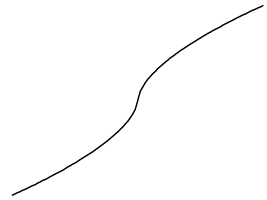
3. Find the line tangent to the curve $y^3 = x^2 - xy$ at $(x,y) = (-2,2)$. Its y-intercept is

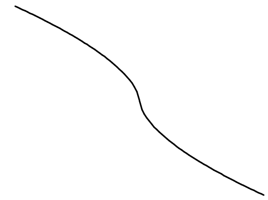
a. $-\frac{4}{5}$
b. $-\frac{3}{5}$
c. $\frac{3}{5}$
d. $\frac{4}{5}$
e. 2

4. Near $x = 2$, the graph of $f(x) = (x-2)^{3/5}$ has the shape:

a. 

b. 

c. 

d. 

e None of these

5. $\lim_{x \to 0} \dfrac{x\cos x - x}{\sin x - x} =$

a. -3
b. $-\dfrac{3}{2}$
c. $\dfrac{1}{2}$
d. $\dfrac{3}{2}$
e. 3

6. When a dead body was found, its temperature was $85°F$. After 2 hours its temperature dropped to $80°F$ while the ambient temperature was $75°F$. How long before the victim was found was its temperature $95°F$.

a. 6 hours
b. 4 hours
c. 2 hours
d. 1 hour
e. $.5$ hours

7. Find all horizontal asymptotes of $f(x) = \tanh(x)$.

a. $y = 1$ and $y = -1$ only
b. $y = 1$ only
c. $y = \frac{\pi}{2}$ only
d. $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$ only
e. $y = -\frac{\pi}{2}$ only

8. If the initial position and velocity of a car are $x(0) = 2$ and $v(0) = 3$ and its accereration is $a(t) = e^{-t} + 6t$ find its position at $t = 2$.

a. $-e^{-2} + 16$
b. $-e^{-2} + 17$
c. $e^{-2} + 13$
d. $e^{-2} + 16$
e. $e^{-2} + 17$

9. Write the integral $\int_2^5 \sqrt{4 + x}\, dx$ as a limit of Riemann sums with intervals of equal length and right endpoints.

a. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \sqrt{4 + \frac{5i}{n}}$
b. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \sqrt{6 + \frac{3i}{n}}$
c. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \sqrt{6 + \frac{5i}{n}}$
d. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \sqrt{4 + \frac{3i}{n}}$
e. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n} \sqrt{6 + \frac{5i}{n}}$

10. Find the points of inflection of the function $f(x) = \frac{1}{4}x^5 - \frac{5}{4}x^4$.

a. $x = 3$ only
b. $x = 0$ and $x = 3$ only
c. $x = -\sqrt{3}$ and $x = \sqrt{3}$ only
d. $x = 0$ only
e. $x = -3$ and $x = 0$ only

11. Evaluate $\int_0^2 \pi + \sqrt{4 - x^2}\, dx$

a. π
b. 2π
c. 3π
d. 4π
e. 6π

12. Find the area under $y = \sqrt{x}$ between $x = 1$ and $x = 4$.

a. $\frac{21}{2}$
b. $\frac{27}{2}$
c. 4
d. $\frac{14}{3}$
e. 6

Part 2 – Work Out Problems (4 questions. Points indicated. No Calculators)

Solve each problem in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

13. (10 points) Let $f(x) = \sinh(x)$ and $g(x) = \operatorname{arcsinh}(x)$ which is the inverse function of $f(x)$.

a. Find $g'\left(\dfrac{e - \frac{1}{e}}{2}\right)$.

b. Find a formula for $\operatorname{arcsinh}(x)$ which does not involve trig or hyperbolic trig functions or their inverses. The answer can only involve $+$ - * / ^ $\sqrt[n]{\ }$ log.
HINT: Write the equation $y = \sinh(x)$ as a quadratic equation for $z = e^x$. Solve for z and then for x.

14. (30 points) Analyze the graph of the function $f(x) = \dfrac{\ln(x)}{x}$. Find each of the following. If none or nowhere or undefined, say so.

a. $\displaystyle \lim_{x \to 0^+} \frac{\ln(x)}{x} =$

b. $\displaystyle \lim_{x \to \infty} \frac{\ln(x)}{x} =$

c. x-intercepts:

d. y-intercepts:

e. horizontal asymptotes:

f. vertical asymptotes:

g. $f'(x) =$

h. $\displaystyle \lim_{x \to 0^+} f'(x) =$

i. $\displaystyle \lim_{x \to \infty} f'(x) =$

j. intervals where increasing:

k. intervals where decreasing:

l. local maxima $\left(x \text{ and } y \text{ coordinates}\right)$:

m. local minima $\left(x \text{ and } y \text{ coordinates}\right)$:

CONTINUED

n. $f''(x) =$

o. $\lim_{x \to 0^+} f''(x) =$

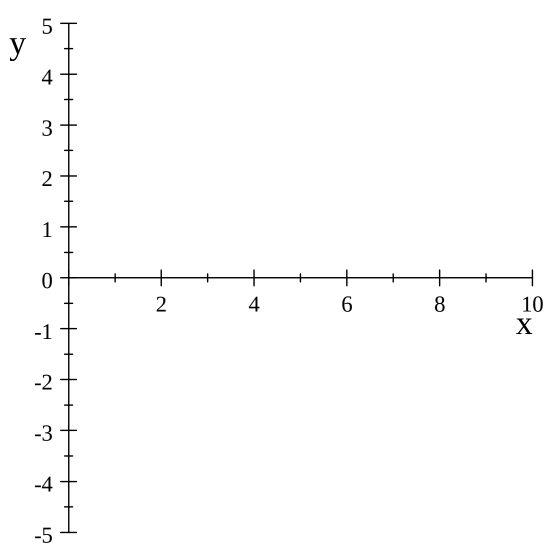
p. $\lim_{x \to \infty} f''(x) =$

q. intervals where concave down:

r. intervals where concave up:

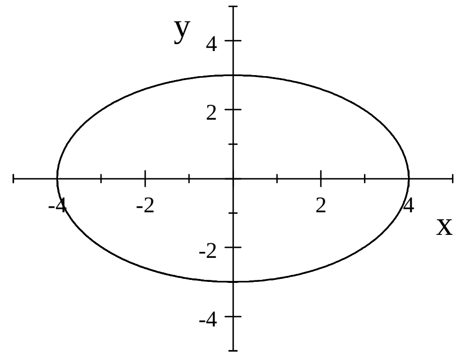
s. inflection points $\left(x \text{ and } y \text{ coordinates}\right)$:

t. Roughly graph the function:



15. (5 points) The ellipse whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ crosses the x-axis at $x = \pm a$ and the y-axis at $y = \pm b$ and has area $A = \pi ab$.

For example, the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with $a = 4$ cm and $b = 3$ cm is shown at the right and has area $A = 12\pi$.



If a is currently 4 cm and decreasing at $\frac{da}{dt} = -2 \ \frac{\text{cm}}{\text{min}}$,

while b is currently 3 cm and increasing at $\frac{db}{dt} = 1 \ \frac{\text{cm}}{\text{min}}$,

is the area increasing or decreasing and at what rate?

16. (10 points) A paint can needs to hold a liter of paint (1000 cm^3). The shape will be a cylinder of radius r and height h. The sides and bottom will be made from aluminum which costs 15¢ per cm^2. The top will be made from plastic which costs 5¢ per cm^2. What are the DIMENSIONS and COST of the cheapest such can? (Keep answers in terms of π.)