MATH 151, FALL 2013
COMMON EXAM I - VERSION B

LAST NAME: Key ______________ FIRST NAME: ______________

INSTRUCTOR: ______________

SECTION NUMBER: ______________

UIN: ______________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-22), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		6
17		8
18		5
19		10
20		10
21		8
22		8
		100

PART I: Multiple Choice. 3 points each

1. Find $\lim_{x \to -1^-} \frac{x-2}{x+1}$ $= \infty$, since $\frac{x-2}{x+1} > 0$ for $x \to 1^-$

   (a) $-\infty$
   (b) 1
   (c) 0
   (d) ∞
   (e) The limit does not exist

2. Given the points $A(0,1)$, $B(2,0)$ and $C(3,-4)$, find the angle, α, located at the vertex A. That is, $\angle BAC$.

   (a) $\alpha = \arccos\left(\frac{-6}{\sqrt{85}}\right)$ $AB = \langle 2, -1 \rangle$
   (b) $\alpha = \arccos\left(-\frac{11}{\sqrt{170}}\right)$ $AC = \langle 3, -5 \rangle$
   (c) $\alpha = \arccos\left(\frac{1}{\sqrt{170}}\right)$ $\cos\alpha = \frac{\langle 2,-1\rangle \cdot \langle 3,-5\rangle}{|\langle 2,-1\rangle|\,|\langle 3,-5\rangle|} = \frac{11}{\sqrt{5}\sqrt{34}}$
   (d) $\alpha = \arccos\left(\frac{11}{\sqrt{170}}\right)$
   (e) $\alpha = \arccos\left(\frac{2}{\sqrt{85}}\right)$

   $\alpha = \arccos\left(\frac{11}{\sqrt{170}}\right)$

3. Find $\lim_{t\to 4} \mathbf{r(t)}$ where $\mathbf{r(t)} = \left\langle 2t+1, \frac{\sqrt{t+5}-3}{t-4} \right\rangle$.

   (a) $\langle 9, 0 \rangle$
   (b) $\left\langle 9, \frac{1}{6} \right\rangle$ $\lim_{t\to4}(2t+1) = 9$; $\lim_{t\to4} \frac{\sqrt{t+5}-3}{t-4} \cdot \frac{\sqrt{t+5}+3}{\sqrt{t+5}+3}$
   (c) $\langle 9, 1 \rangle$
   (d) $\left\langle 9, -\frac{1}{6} \right\rangle$ $\left\langle 9, \frac{1}{6} \right\rangle$ $\lim_{t\to4} \frac{\cancel{t+5-9}}{\cancel{(t-4)}(\sqrt{t+5}+3)} = \frac{1}{6}$
   (e) $\left\langle 9, -\frac{1}{2} \right\rangle$

4. Find the vector projection of $\langle -3, 1 \rangle$ onto $\langle 2, 5 \rangle$.

   (a) $\left\langle -\frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}} \right\rangle$ $\vec{a} = \langle 2, 5 \rangle$ $\vec{b} = \langle -3, 1 \rangle$
   (b) $\left\langle \frac{3}{10}, -\frac{1}{10} \right\rangle$
   (c) $\left\langle -\frac{2}{29}, -\frac{5}{29} \right\rangle$ $\text{proj}_a b = \frac{\langle 2,5\rangle \cdot \langle -3,1\rangle}{|\langle 2,5\rangle|^2} \langle 2,5 \rangle$
   (d) $\left\langle \frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \right\rangle$
   (e) $\left\langle \frac{11}{29}, \frac{55}{29} \right\rangle$

   $= -\frac{1}{29} \langle 2, 5 \rangle$

5. If $\mathbf{a} = \langle 1, 1 \rangle$, $\mathbf{b} = \langle 2, 1 \rangle$ and $\mathbf{c} = \langle 4, -3 \rangle$, what value of t satisfies $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$, where s and t are scalars?

(a) $t = 2$
(b) $t = 7$
(c) $t = -10$
(d) $t = 1$
(e) $t = -7$

$\langle 4, -3 \rangle = s\langle 1, 1 \rangle + t\langle 2, 1 \rangle$

$\langle 4, -3 \rangle = \langle s + 2t, s + t \rangle$

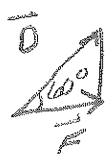
$s + 2t = 4 \to s = 4 - 2t$

$s + t = -3 \to 4 - 2t + t = -3$

$-t = -7 \quad \boxed{t = 7}$

6. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of 60° above the horizontal. Find the work done on the box.

(a) 50 foot pounds
(b) $50\sqrt{2}$ foot pounds
(c) 100 foot pounds
(d) 10 foot pounds
(e) $50\sqrt{3}$ foot pounds



$W = |F||D| \cos\theta$

$= (20 \text{ lbs})(5 \text{ ft}) \cos 60°$

$= (100)(\frac{1}{2}) \text{ ft lbs}$

7. Which interval contains a solution to the equation $x^3 + x = 3$?

(a) $[-1, 0]$
(b) $[-2, -1]$
(c) $[0, 1]$
(d) $[0, 2]$
(e) $[2, 3]$

Let $f(x) = x^3 + x$, a continuous function.

Since $f(0) = 0 < 3$

$f(2) = 10 > 3$

a solution to $f(x) = 3$ exists on $[0, 2]$

8. Consider $f(x) = \begin{cases} x^2 + 5x + 1 & \text{if } x < -1 \\ 3 & \text{if } x = -1 \\ 2x - 1 & \text{if } x > -1 \end{cases}$. Why is $f(x)$ not continuous at $x = -1$?

(a) $f(x)$ is not continuous at $x = -1$ because $f(-1)$ does not exist.
(b) $f(x)$ is not continuous at $x = -1$ because $\lim\limits_{x \to -1} f(x) \neq f(-1)$.
(c) $f(x)$ is not continuous at $x = -1$ because $\lim\limits_{x \to -1} f(x)$ does not exist.
(d) $f(x)$ is not continuous at $x = -1$ because $\lim\limits_{x \to -1^+} f(x)$ does not exist.
(e) $f(x)$ is not continuous at $x = -1$ because $\lim\limits_{x \to -1^-} f(x)$ does not exist.

$\lim\limits_{x \to -1} f(x) = -3$, but $f(-1) = 3$

9. Find $\lim_{x\to 3^-} \frac{|x-3|}{x^2-2x-3}$

(a) $\frac{1}{4}$

(b) 0

(c) ∞

(d) $-\frac{1}{4}$

(e) The limit does not exist

if $x < 3$, $|x-3| = -(x-3)$

$$\lim_{x\to 3^-} \frac{|x-3|}{x^2-2x-3} = \lim_{x\to 3^-} \frac{-\cancel{(x-3)}}{\cancel{(x-3)}(x+1)}$$

$$= -\frac{1}{4}$$

10. Find $\lim_{x\to 2} \frac{x^4-16}{x-2}$.

(a) 1

(b) ∞

(c) 32

(d) 4

(e) 0

$$= \lim_{x\to 2} \frac{(x^2-4)(x^2+4)}{x-2}$$

$$= \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}(x^2+4)}{\cancel{x-2}}$$

$$= (4)(8)$$

11. Find the average rate of change of $f(t) = \sqrt{2t+3}$ from $t = 1$ to $t = 3$.

(a) $3-\sqrt{5}$

(b) $\frac{3-\sqrt{5}}{2}$

(c) $\frac{\sqrt{5}-3}{2}$

(d) $3+\sqrt{5}$

(e) $\frac{3+\sqrt{5}}{2}$

$$AROC = \frac{f(3)-f(1)}{3-1}$$

$$= \frac{3-\sqrt{5}}{2}$$

12. Find the horizontal and vertical asymptotes for $f(x) = \frac{(2-x)(3x+1)}{x^2-4}$.

(a) $y = -3$, $x = -2$

(b) $y = -3$, $x = 2$, $x = -2$

(c) $x = -3$, $y = 2$, $y = -2$

(d) $x = -3$, $y = -2$

(e) $y = 3$, $x = -2$

$$= \frac{(2-x)(3x+1)}{(x+2)(x-2)}$$

horizontal asymptote $y = -3$

vertical asymptote $x = -2$

13. Find a unit vector in the direction of $\mathbf{a} - \mathbf{b}$ where $\mathbf{a} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = -5\mathbf{j}$.

(a) $\frac{6}{\sqrt{45}}\mathbf{i} - \frac{3}{\sqrt{45}}\mathbf{j}$

(b) $\frac{8}{\sqrt{65}}\mathbf{i} - \frac{1}{\sqrt{65}}\mathbf{j}$

(c) $\frac{1}{\sqrt{65}}\mathbf{i} - \frac{2}{\sqrt{65}}\mathbf{j}$

(d) $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$

(e) $\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$

$\vec{a} - \vec{b} = \langle 1, -3 \rangle - \langle 0, -5 \rangle$

$= \langle 1, 2 \rangle$

$\vec{u} = \frac{\langle 1, 2 \rangle}{|\langle 1, 2 \rangle|} = \frac{\langle 1, 2 \rangle}{\sqrt{5}} = \frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$

14. Find the value of x so that the vectors $\langle 4, x+1 \rangle$ and $\langle x, 3 \rangle$ are perpendicular.

(a) $x = 0$

(b) $x = -\frac{3}{7}$

(c) $x = -\frac{7}{3}$

(d) $x = -\frac{1}{7}$

(e) $x = \frac{1}{7}$

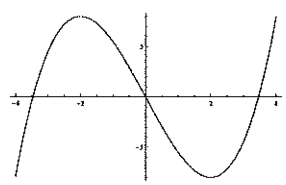
$\langle 4, x+1 \rangle \cdot \langle x, 3 \rangle = 0$

$4x + 3(x+1) = 0$

$7x = -3$

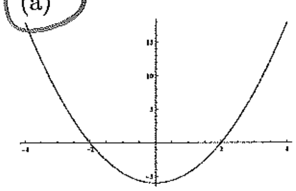
$x = -\frac{3}{7}$

15. Consider The graph of $f(x)$ given:

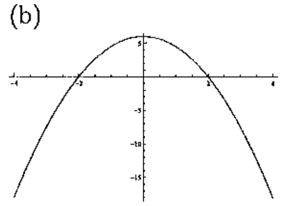


Which of the following is the graph of its derivative, $f'(x)$?

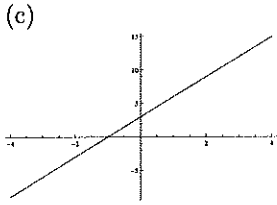
(a)



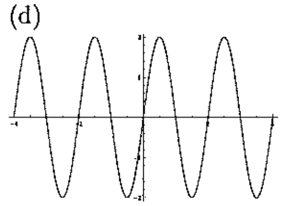
(b)



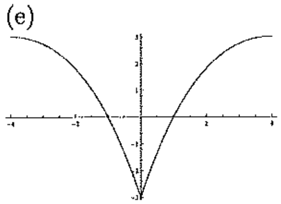
(c)



(d)



(e)



PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Consider the line $x = 6 + 3t$, $y = 10 - 5t$.

(i) (2 pts) Find a vector parallel to the line.

$\vec{v} = \langle 3, -5 \rangle$ (or any scalar multiple of this vector)

(ii) (2 pts) Find a vector perpendicular to the line.

$\langle 5, 3 \rangle$ (or any scalar multiple of this vector)

(iii) (2 pts) Find the x and y intercepts of the line.

x-int: $y = 0 \to 0 = 10 - 5t \to t = 2 \to \boxed{x = 12}$

y-int: $x = 0 \to 0 = 6 + 3t \to t = -2 \to \boxed{y = 20}$

17. (8 pts) Find $\lim_{x\to\infty} \left(\sqrt{x^2+8x-1} - x\right)$.

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+8x-1} - x\right)\left(\sqrt{x^2+8x-1} + x\right)}{\sqrt{x^2+8x-1} + x}$$

$$\lim_{x\to\infty} \frac{x^2+8x-1-x^2}{\sqrt{x^2+8x-1}+x}$$

$$\lim_{x\to\infty} \frac{8x-1}{\sqrt{x^2+8x-1}+x}$$

$$\to \lim_{x\to\infty} \frac{\frac{8x-1}{x}}{\frac{\sqrt{x^2+8x-1}+x}{x}}$$

$$\lim_{x\to\infty} \frac{8-\frac{1}{x}}{\sqrt{1+\frac{8}{x}-\frac{1}{x^2}}+1}$$

$$= \frac{8}{2} = 4$$

18. (5 pts) If $f(3) = 4$ and $f'(3) = -9$, find the equation of the tangent line to the graph of $f(x)$ at $x = 3$.

$m = f'(3) = -9$

point $(3, 4)$

$y - 4 = -9(x - 3)$

or $y = -9x + 31$

19. (10 pts) For $f(x) = \dfrac{1}{3x+1}$, find $f'(x)$ using the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3x+3h+1} - \frac{1}{3x+1}}{h}$$

$$= \lim_{h \to 0} \frac{3x+1-(3x+3h+1)}{h(3x+3h+1)(3x+1)}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(3x+3\underset{0}{\underset{\downarrow}{h}}+1)(3x+1)}$$

$$= \frac{-3}{(3x+1)^2}$$

20. (10 pts) Consider $f(x) = \begin{cases} cx+2 & \text{if } x > 6 \\ \frac{1}{5} & \text{if } x = 6 \\ cx^2 - 4 & \text{if } x < 6 \end{cases}$

(i) Find $\lim_{x \to 6^+} f(x)$ in terms of c.

$\lim_{x \to 6^+} f(x) = 6c + 2$

(ii) Find $\lim_{x \to 6^-} f(x)$ in terms of c.

$\lim_{x \to 6^-} f(x) = 36c - 4$

(iii) For what value of c does $\lim_{x \to 6} f(x)$ exist?

$6c + 2 = 36c - 4$

$6 = 30c$

$c = \frac{1}{5}$

(iv) For the value of c found above, what is $\lim_{x \to 6} f(x)$?

$f(x) = \begin{cases} \frac{1}{5}x + 2, & x > 6 \\ \frac{1}{5} & x = 6 \\ \frac{1}{5}x^2 - 4 & x < 6 \end{cases}$

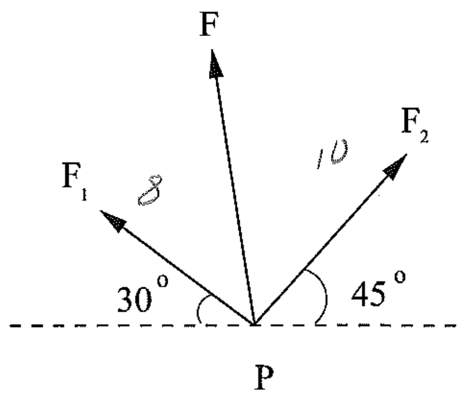
$\lim_{x \to 6} f(x) = \frac{6}{5} + 2$

$= \frac{16}{5}$

(v) For the value of c above, is $f(x)$ continuous at $x = 6$? Support your answer.

No, because $\lim_{x \to 6} f(x) = \frac{16}{5} \neq f(6)$

21. (8 pts) Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ with magnitudes 8 lbs and 10 lbs, respectively, act on an object at a point P as shown.



(i) Find the vector, $\mathbf{F_1}$. Evaluate trig functions.

$$F_1 = \langle -8 \cdot \tfrac{\sqrt{3}}{2},\ 8 \cdot \tfrac{1}{2} \rangle$$

$$\boxed{F_1 = \langle -4\sqrt{3},\ 4 \rangle}$$

(ii) Find the vector, $\mathbf{F_2}$. Evaluate trig functions.

$$F_2 = \langle 10\ \tfrac{\sqrt{2}}{2},\ 10\ \tfrac{\sqrt{2}}{2} \rangle$$

$$\boxed{F_2 = \langle 5\sqrt{2},\ 5\sqrt{2} \rangle}$$

(iii) Find the resultant force, $\mathbf{F}$, acting on the object.

$$F = F_1 + F_2$$

$$\boxed{F = \langle -4\sqrt{3} + 5\sqrt{2},\ 4 + 5\sqrt{2} \rangle}$$

22. Consider the curve $x = -2 + \cos t$, $y = 3 + \sin t$.

(i) (4 pts) Eliminate the parameter to find a Cartesian equation.

$$x + 2 = \cos t$$

$$y - 3 = \sin t$$

$$(x+2)^2 + (y-3)^2 = 1$$

(ii) (4 pts) Sketch the curve on the grid below.