MATH 151, FALL 2013
COMMON EXAM II - VERSION A

LAST NAME: Key FIRST NAME: ____________________

INSTRUCTOR: ____________________

SECTION NUMBER: __________

UIN: ____________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-21), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		10
17		6
18		12
19		12
20		8
21		7
		100

PART I: Multiple Choice. 3 points each

1. Find $\lim_{t\to 0} \frac{\sin^2(6t)}{4t^2}$. $= \lim_{t\to 0} \frac{\sin^2(6t)}{t^2}\cdot\frac{1}{4}$

   (a) $\frac{3}{2}$
   (b) 9
   (c) $\frac{1}{9}$
   (d) 0
   (e) ∞

$= \frac{1}{4}\lim_{t\to 0}\left(\frac{\sin(6t)}{t}\right)^2\cdot\frac{36}{36}$

$= \frac{36}{4}\lim_{t\to 0}\left(\frac{\sin(6t)}{6t}\right)^2 = \boxed{9}$

2. If $g(x) = xf(x^3)$, $f(2) = 4$, $f(8) = 3$, $f'(8) = -1$, and $f'(2) = -2$, what is $g'(2)$?

   (a) -24
   (b) -12
   (c) -16
   (d) -21
   (e) -44

$g'(x) = f(x^3) + x f'(x^3)\cdot 3x^2$

$g'(2) = f(8) + 2\cdot f'(8)\cdot 12$

$= 3 + 2(-1)(12) = 3-24$

$= \boxed{-21}$

3. Find the slope of the tangent line to the curve $x^3 + y^3 = x^2 + 5y$ at the point $(2, 1)$.

   (a) $m = 4$
   (b) $m = -1$
   (c) $m = \frac{11}{5}$
   (d) $m = 2$
   (e) $m = \frac{1}{4}$

$3x^2 + 3y^2\frac{dy}{dx} = 2x + 5\frac{dy}{dx}$

$\frac{dy}{dx}(3y^2 - 5) = 2x - 3x^2$

$\frac{dy}{dx} = \frac{2x-3x^2}{3y^2-5} \rightarrow m = \frac{4-12}{-2}$

$\boxed{m = 4}$

4. Use differentials to approximate $\sqrt[3]{8.2}$.

   (a) $\sqrt[3]{8.2} \approx \frac{61}{60}$
   (b) $\sqrt[3]{8.2} \approx \frac{22}{5}$
   (c) $\sqrt[3]{8.2} \approx \frac{119}{60}$
   (d) $\sqrt[3]{8.2} \approx \frac{11}{5}$
   (e) $\sqrt[3]{8.2} \approx \frac{121}{60}$

Let $f(x) = \sqrt[3]{x}$

$a = 8,\ dx = 0.2$

$\sqrt[3]{8.2} \approx f(8) + f'(8)\,dx$

$\approx 2 + \frac{1}{12}(0.2)$

$\approx 2 + \frac{1}{60} = \frac{121}{60}$

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f(8) = 2$

$f'(8) = \frac{1}{3}(8)^{-\frac{2}{3}}$

$= \frac{1}{12}$

5. Find $\lim_{x\to\infty} \frac{2^x + 2^{-4x}}{7(2^x) - 2^{-4x}}$.

(a) $\frac{2}{7}$

(b) $\frac{1}{7}$

(c) ∞

(d) 0

(e) -1

$$= \lim_{x\to\infty} \frac{2^x + \frac{1}{2^{4x}}}{7(2^x) - \frac{1}{2^{4x}}} \left(\frac{\frac{1}{2^x}}{\frac{1}{2^x}}\right)$$

$$= \lim_{x\to\infty} \frac{1 + \frac{1}{2^{5x}}\to 0}{7 - \frac{1}{2^{5x}}\to 0} = \frac{1}{7}$$

6. The function $f(x) = 4x + \sin x$ is a one-to-one function. If g is the inverse of f, what is $g'(4\pi)$?

(a) $-\frac{1}{5}$

(b) -1

(c) $\frac{1}{5}$

(d) $\frac{1}{4}$

(e) $\frac{1}{3}$

$f(\pi) = 4\pi$, thus $g(4\pi) = \pi$

$$g'(4\pi) = \frac{1}{f'(g(4\pi))}$$

$f'(x) = 4 + \cos x$

$f'(\pi) = 3$

$$= \frac{1}{f'(\pi)} = \boxed{\frac{1}{3}}$$

7. Consider the parametric curve $x(t) = t^4 + 1$ and $y(t) = \cos\left(\frac{\pi}{2}t\right)$. Find the slope of the tangent line at the point $(2,0)$.

(a) $-\frac{\pi}{8}$

(b) $-\frac{1}{4}$

(c) $-\frac{8}{\pi}$

(d) 0

(e) $-\frac{\pi}{4}$

if $t=1$, $x=2$, $y=0$

$$m = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = \left.\frac{-\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)}{4t^3}\right|_{t=1}$$

$$= \frac{-\frac{\pi}{2}}{4} = \boxed{\frac{-\pi}{8}}$$

8. Find $h''(1)$ if $h(x) = e^{-x^2}$.

(a) $\frac{4}{e}$

(b) $\frac{2}{e}$

(c) $-\frac{2}{e}$

(d) $\frac{1}{e}$

(e) $-\frac{4}{e}$

$h'(x) = -2xe^{-x^2}$

$h''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$

$h''(1) = -2e^{-1} + 4e^{-1}$

$= 2e^{-1}$ or $\frac{2}{e}$

9. $\lim_{x \to 2^-} \left(\frac{1}{4}\right)^{\frac{x}{x-2}}$

(a) 0
(b) $-\infty$
(c) $\frac{1}{4}$
(d) 1
(e) ∞

$\left(\frac{1}{4}\right)^{\lim_{x\to 2^-} \frac{x}{x-2}} = \left(\frac{1}{4}\right)^{-\infty}$

$= \infty$

10. At what point on the curve $x = 2t^2 - 4t$, $y = t^2 - t$ is the tangent line horizontal?

(a) $(-1, 0)$
(b) $\left(\frac{1}{2}, -\frac{1}{4}\right)$
(c) $(0, 0)$
(d) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$
(e) $(-2, 0)$

$\frac{dy}{dt} = 0 \to 2t - 1 = 0$

$\to t = \frac{1}{2}$

$x = 2\left(\frac{1}{4}\right) - 2 = -\frac{3}{2}$

$y = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

point $= \left(-\frac{3}{2}, -\frac{1}{4}\right)$

11. Find the equation of the tangent line to the curve $g(x) = \dfrac{x}{2x+1}$ at $x = 4$.

(a) $y - \frac{4}{9} = \frac{1}{2}(x-4)$
(b) $y - \frac{4}{9} = \frac{1}{81}(x-4)$
(c) $y = \frac{1}{81}(x-4)$
(d) $y - \frac{4}{3} = \frac{1}{81}(x-4)$
(e) $y - \frac{4}{9} = \frac{1}{9}(x-4)$

$g'(x) = \frac{2x+1-2x}{(2x+1)^2} = \frac{1}{(2x+1)^2}$

$m = g'(4) = \frac{1}{81}$ point $\left(4, \frac{4}{9}\right)$

$y - \frac{4}{9} = \frac{1}{81}(x-4)$

12. Find the quadratic approximation for $f(x) = e^{4x}$ at $x = 0$.

(a) $Q(x) = 1 + 4x + 16x^2$
(b) $Q(x) = 1 + x + \frac{1}{2}x^2$
(c) $Q(x) = 1 + x + x^2$
(d) $Q(x) = 1 + 4x + 8x^2$
(e) $Q(x) = 1 + 4x + 2x^2$

$f'(x) = 4e^{4x}$

$f''(x) = 16e^{4x}$

$Q(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$

$= 1 + 4x + 8x^2$

13. An object is moving according to the equation of motion $s(t) = \cos t + \frac{1}{4}t^2$. Find the time(s) when the acceleration is zero for $0 \leq t \leq 2\pi$.

(a) $t = \frac{2\pi}{3}, t = \frac{4\pi}{3}$

(b) $t = \frac{\pi}{6}, t = \frac{11\pi}{6}$

(c) $t = \frac{5\pi}{6}, t = \frac{7\pi}{6}$

(d) $t = \frac{\pi}{4}, t = \frac{7\pi}{4}$

(e) $t = \frac{\pi}{3}, t = \frac{5\pi}{3}$

$v(t) = -\sin t + \frac{t}{2}$

$a(t) = -\cos t + \frac{1}{2}$

$a(t) = 0 \rightarrow -\cos t + \frac{1}{2} = 0$

$\rightarrow \cos t = \frac{1}{2} \quad t = \frac{\pi}{3}, \frac{5\pi}{3}$

14. Find the value(s) of x where the tangent line to the graph of $f(x) = x\sqrt{x}$ is parallel to the line $2x - 4y = 18$.

(a) $x = 9$

(b) $x = \pm 9$

(c) $x = \frac{16}{3}$

(d) $x = \frac{1}{9}$

(e) $x = \pm\frac{1}{9}$

$2x - 4y = 18 \rightarrow 4y = 2x - 18$

$y = \frac{1}{2}x - \frac{18}{4} \rightarrow m = \frac{1}{2}$

solve $f'(x) = \frac{1}{2}$

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} \rightarrow \frac{3}{2}\sqrt{x} = \frac{1}{2}$

$\sqrt{x} = \frac{1}{3}$

$x = \frac{1}{9}$

15. Find the unit tangent vector to the curve $\mathbf{r(t)} = \langle \sin(3t), \cos(3t) \rangle$ at $t = \frac{\pi}{9}$.

(a) $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

(b) $\left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

(c) $\left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$

(d) $\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

(e) $\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

$r'(t) = \langle 3\cos(3t), -3\sin(3t) \rangle$

$r'(\frac{\pi}{9}) = \langle 3 \cdot \frac{1}{2}, -3 \cdot \frac{\sqrt{3}}{2} \rangle$ divide by magnitude

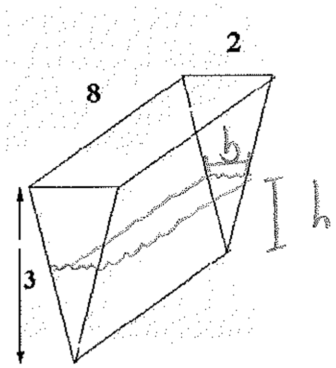
$|r'(9)| = \sqrt{\frac{9}{4} + \frac{27}{4}} = \frac{6}{2} = 3$

$\vec{u} = \frac{\langle \frac{3}{2}, -\frac{3\sqrt{3}}{2} \rangle}{3} = \langle \frac{1}{2}, \frac{-\sqrt{3}}{2} \rangle$

PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (10 pts) A trough is 8 feet long. The ends of the trough are isosceles triangles with height 3 feet and width 2 feet across the top. If water is being poured into the trough at a rate of 2 cubic feet per minute, how fast is the water level rising when the depth of the water is 1 foot?



given: $\frac{dV}{dt} = 2 \frac{ft^3}{min}$

Find $\left.\frac{dh}{dt}\right|_{h=1 \text{ foot}}$

$$V = \left(\tfrac{1}{2} bh\right)(8)$$

$$V = 4bh \qquad \frac{b}{h} = \frac{2}{3}$$

$$b = \frac{2}{3}h$$

$$V = 4\left(\frac{2}{3}h\right)h$$

$$V = \frac{8}{3}h^2$$

$$\frac{dV}{dt} = \frac{16}{3}h\frac{dh}{dt}$$

$$2 = \frac{16}{3}(1)\frac{dh}{dt}$$

$$2 = \frac{16}{3}\frac{dh}{dt}$$

$$\frac{dh}{dt} = 2 \cdot \frac{3}{16}$$

$$\boxed{\frac{dh}{dt} = \frac{3}{8}\frac{f}{m}}$$

17. (6 pts) Find $\frac{dy}{dx}$ if $y = e^{3xy}$.

$$\frac{dy}{dx} = \left(3y + 3x\frac{dy}{dx}\right)e^{3xy}$$

$$\frac{dy}{dx} = 3ye^{3xy} + 3xe^{3xy}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(1 - 3xe^{3xy}\right) = 3ye^{3xy}$$

$$\boxed{\frac{dy}{dx} = \frac{3ye^{3xy}}{1-3xe^{3xy}}}$$

18. Find the derivative of the following functions. Do not simplify.

a.) (4 pts) $f(x) = \left((x^2+3)^5 + x\right)^8$

$$f'(x) = 8\left((x^2+3)^5 + x\right)^7\left(5(x^2+3)^4(2x) + 1\right)$$

$$= 8\left((x^2+3)^5 + x\right)^7\left(10x(x^2+3)^4 + 1\right)$$

b.) (4 pts) $g(x) = \tan\left(\sqrt{x^2+3x}\right)$

$$g'(x) = \sec^2\left(\sqrt{x^2+3x}\right) \cdot \frac{1}{2}(x^2+3x)^{-\frac{1}{2}}(2x+3)$$

$$= \frac{\sec^2\sqrt{x^2+3x}\,(2x+3)}{2\sqrt{x^2+3x}}$$

c.) (4 pts) $h(x) = \cos^4\left(a^3 + \frac{1}{x^2}\right)$

$$h'(x) = 4\cos^3\left(a^3 + \frac{1}{x^2}\right)\left[-\sin\left(a^3+\frac{1}{x^2}\right) \cdot \frac{-2}{x^3}\right]$$

19. Given that $h(5) = 3$, $h'(5) = -2$, $g(5) = -3$ and $g'(5) = 6$, find $f'(5)$ for each of the following, if possible. If it is not possible, state what additional information is required.

a.) (3 pts) $f(x) = g(x)h(x)$

$$f'(x) = g(x)h'(x) + g'(x)h(x)$$

$$f'(5) = g(5)h'(5) + g'(5)h(5)$$

$$= (-3)(-2) + (6)(3) = 6 + 18$$

$$= 24$$

b.) (3 pts) $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(5) = \frac{(6)(3) - (-3)(-2)}{9} = \frac{18 - 6}{9} = \frac{12}{9} = \frac{4}{3}$$

c.) (3 pts) $f(x) = [g(x)]^3$

$$f'(x) = 3[g(x)]^2 g'(x)$$

$$f'(5) = 3[-3]^2 6$$

$$= 27(6) = 162$$

d.) (3 pts) $f(x) = g(h(x))$

$$f'(x) = g'(h(x))h'(x)$$

$$f'(5) = g'(h(5))h'(5)$$

$$= g'(3)(-2)$$

not possible. we do not know $g'(3)$.

20. Consider the vector equation $\mathbf{r}(\mathbf{t}) = \left\langle \cos\left(\frac{1}{t}\right), \sqrt{4-t^2} \right\rangle$.

a.) (2 pts) What is the domain of $\mathbf{r}(\mathbf{t})$? Use interval notation.

$t \neq 0, \quad -2 \leq t \leq 2$

$[-2, 0) \cup (0, 2]$

b.) (6 pts) Find $\mathbf{r}'(\mathbf{t})$ and the domain of $\mathbf{r}'(\mathbf{t})$. Use interval notation.

$r'(t) = \left\langle -\sin\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right), \frac{1}{2}(4-t^2)^{-\frac{1}{2}}(-2t) \right\rangle$

$= \left\langle \frac{1}{t^2}\sin\left(\frac{1}{t}\right), \frac{-t}{\sqrt{4-t^2}} \right\rangle$

domain $r'(t)$

$(-2, 0) \cup (0, 2)$

21. Consider $f(x) = \begin{cases} ax^2 + x + 1 & \text{if } x \leq -1 \\ bx - 1 & \text{if } x > -1 \end{cases}$.

a.) (4 pts) Find the value of a and b that make $f(x)$ differentiable everywhere.

continuous: $a - 1 + 1 = -b - 1 \rightarrow a = -b - 1$

$f'(x) = \begin{cases} 2ax + 1 & x \leq -1 \\ b & x > -1 \end{cases}$

differentiable $\quad -2a + 1 = b \rightarrow -2(-b-1) + 1 = b$

$2b + 2 + 1 = b$

$b = -3$

$a = 2$

b.) (3 pts) For the value of a and b found above, find $f'(x)$.

$f'(x) = \begin{cases} 4x + 1 & x \leq -1 \\ -3 & x > -1 \end{cases}$