MATH 151, FALL 2013
COMMON EXAM III - VERSION A

LAST NAME: Key FIRST NAME: ______

INSTRUCTOR: ______

SECTION NUMBER: ______

UIN: ______

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-21), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

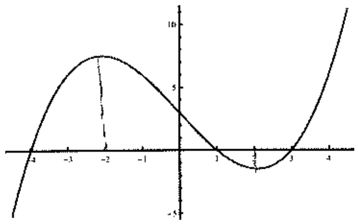
Signature: ______

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		5
17		7
18		16
19		8
20		6
21		8
22		5
		100

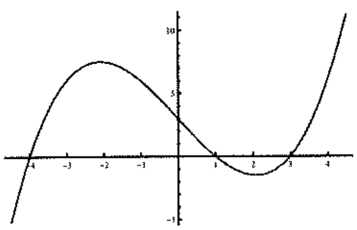
PART I: Multiple Choice. 3 points each

1. If the graph below is that of the derivative, $f'(x)$, of a function $f(x)$, what are the x coordinates of the inflection point(s) of $f(x)$?



(a) $x = 0$
(b) $x = -4$, $x = 1$, $x = 3$
(c) $x = 1$
(d) $x = \pm 2$
(e) There are no points of inflection

2. If the graph below is that of the derivative, $f'(x)$, of a function $f(x)$, where is $f(x)$ decreasing?



(a) $(-4, 1) \cup (3, \infty)$
(b) $(-\infty, -4) \cup (1, 3)$
(c) $(-2, 2)$
(d) $(-\infty, -2) \cup (2, \infty)$
(e) $(-\infty, 0)$

3. $\lim_{x \to \infty} (\ln(x+2) - \ln(3x+1)) =$

$\lim_{x \to \infty} \ln\left(\frac{x+2}{3x+1}\right)$

$= \ln \frac{1}{3}$

(a) $\ln \frac{1}{3}$
(b) 0
(c) 1
(d) ∞
(e) $-\infty$

4. Given that $f(x)$ is defined everywhere except $x = -3$ and $f'(x) = \dfrac{x^2(1-x)}{(x+3)^3}$, on what intervals is $f(x)$ increasing?

(a) $(-\infty, 0) \cup (1, \infty)$
(b) $(-3, 0) \cup (1, \infty)$
(c) $(-\infty, -3) \cup (1, \infty)$
(d) $(-\infty, 0) \cup (0, 1)$
(e) $(-3, 0) \cup (0, 1)$

$-$ $+$ $+$ $-$

-3 0 1

5. Find $f''(1)$ if $f(x) = \arctan(2x)$.

(a) $-\frac{16}{25}$

(b) $-\frac{8}{9}$

(c) $-\frac{4}{9}$

(d) $-\frac{8}{25}$

(e) $-\frac{2}{25}$

$$f'(x) = \frac{2}{1+4x^2} = 2(1+4x^2)^{-1}$$

$$f''(x) = -2(1+4x^2)^{-2}(8x)$$

$$f''(x) = \frac{-16x}{(1+4x^2)^2} \quad f''(1) = \frac{-16}{25}$$

6. Calculate $\sin(\arctan(2))$.

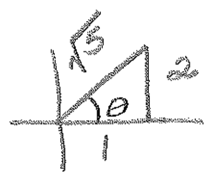
(a) 2

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{2}{\sqrt{5}}$

(d) $\frac{2}{\sqrt{3}}$

(e) $\frac{1}{\sqrt{5}}$



$$\sin\theta = \frac{2}{\sqrt{5}}$$

7. Find $\frac{dy}{dx}$ if $y = (1+2x)^{x^2}$.

(a) $\frac{dy}{dx} = (1+2x)^{x^2}\left(2x\ln(1+2x) + \frac{x^2}{1+2x}\right)$

(b) $\frac{dy}{dx} = (1+2x)^{x^2}\left(2x\ln(1+2x) + \frac{2x^2}{1+2x}\right)$

(c) $\frac{dy}{dx} = \left(2x\ln(1+2x) + \frac{2x^2}{1+2x}\right)$

(d) $\frac{dy}{dx} = (1+2x)^{x^2}\left(\frac{4x}{1+2x}\right)$

(e) $\frac{dy}{dx} = x^2(1+2x)^{x^2-1}(2)$

$$y = (1+2x)^{x^2}$$

$$\ln y = \ln(1+2x)^{x^2}$$

$$\ln y = x^2 \ln(1+2x)$$

$$\frac{1}{y}y' = 2x\ln(1+2x) + x^2 \frac{2}{1+2x}$$

$$y' = y\left[2x\ln(1+2x) + \frac{2x^2}{1+2x}\right]$$

$$y' = (1+2x)^{x^2}\left[2x\ln(1+2x) + \frac{2x^2}{1+2x}\right]$$

8. Find $f'(e^2)$ if $f(x) = \ln(1+\ln x)$.

(a) $\frac{1}{2e^2}$

(b) $\frac{e^2}{1+e^2}$

(c) $\frac{1}{1+e^2}$

(d) $\frac{1}{3e^2}$

(e) $\frac{1}{e^2+2e}$

$$f'(x) = \frac{\frac{1}{x}}{1+\ln x}$$

$$f'(e^2) = \frac{\frac{1}{e^2}}{1+\ln e^2} = \frac{1}{e^2(3)}$$

9. Find the x coordinate where $f(x) = x\sqrt{x+1}$ has a local minimum.

(a) $x = -\frac{1}{3}$
(b) $x = -\frac{2}{3}$
(c) $x = -1$
(d) $x = 0$
(e) None, $f(x)$ is always increasing.

$f'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

$= \frac{2x+2+x}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}}$

$- \quad +$

$-1 \quad -\frac{2}{3}$

10. Find an antiderivative of $f(x) = \frac{x^3 - x^2 + 5}{x^4}$.

(a) $\ln|x| + \frac{1}{x} - \frac{5}{3x^3} + C$
(b) $\ln|x| + \frac{1}{x} - \frac{1}{x^5} + C$
(c) $\ln|x| + \frac{1}{3x^3} - \frac{5}{3x^3} + C$
(d) $\ln|x| + \frac{1}{3x^3} - \frac{1}{x^5} + C$
(e) $\frac{x^0}{0} + \frac{1}{x} - \frac{5}{3x^3} + C$

$f(x) = \frac{1}{x} - \frac{1}{x^2} + \frac{5}{x^4}$

$f(x) = \frac{1}{x} - x^{-2} + 5x^{-4}$

$F(x) = \ln|x| - \frac{x^{-1}}{-1} + \frac{5x^{-3}}{-3} + C$

$= \ln|x| + \frac{1}{x} - \frac{5}{3x^3} + C$

11. Find $\lim_{x \to 1} \frac{e^{2x-2} + x^2 - 2}{\ln(x) + 2x - 2}$.

(a) 0
(b) 1
(c) 2
(d) $\frac{4}{3}$
(e) $\frac{2}{3}$

$\overset{L}{=} \lim_{x \to 1} \frac{2e^{2x-2} + 2x}{\frac{1}{x} + 2}$

$= \frac{4}{3}$

12. Solve $\log_2(x^2 - 38) - \log_2(5 - x) = 1$ for x.

(a) $x = 6$
(b) $x = -8$, $x = 6$
(c) $x = -8$
(d) $x = 8$
(e) No solution

$\log_2 \frac{x^2-38}{5-x} = 1$

$\frac{x^2-38}{5-x} = 2$

$x^2 - 38 = 2(5-x)$

$x^2 - 38 = 10 - 2x$

$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0$

$x = -8$

$\cancel{x = 6}$

13. Find the sum $\sum_{i=2}^{98}\left(\frac{1}{i+1}-\frac{1}{i+2}\right)$. $= \frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\cdots+\frac{1}{99}-\frac{1}{100}$

(a) $\frac{1}{3}$
(b) 0
(c) $\frac{49}{100}$
(d) $\frac{1}{100}$
(e) $\frac{97}{300}$

$= \frac{1}{3}-\frac{1}{100}$

$= \frac{97}{300}$

14. Find the absolute maximum and absolute minimum values for $f(x)=(x^2-1)^3$ on the interval $[-2,2]$.

(a) Absolute maximum: 27. Absolute minimum: 0.
(b) Absolute maximum: 27. Absolute minimum: -1.
(c) Absolute maximum: 108. Absolute minimum: -108.
(d) Absolute maximum: 125. Absolute minimum: -1.
(e) Absolute maximum: 125. Absolute minimum: 0.

$f'(x) = 3(x^2-1)^2(2x)$

$x = 0, \pm 1$

$f(0) = -1$ min

$f(\pm 1) = 0$

$f(\pm 2) = 27$ max

15. Find the inflection point(s) for $f(x)=xe^{2x}$.

(a) $(-\frac{1}{2}, -\frac{1}{2}e^{-1})$
(b) $(-2, -2e^{-4})$
(c) $(0,0)$
(d) $(-\frac{1}{4}, -\frac{1}{4}e^{-1/2})$
(e) $(-1, -e^{-2})$

$f'(x) = e^{2x} + 2xe^{2x}$

$f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x}$

$= 4e^{2x} + 4xe^{2x}$

$f''(x) = 4e^{2x}(1+x)$

CD, CU

$-$ | $+$

-1

infl pt: $(-1, -e^{-2})$

PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (5 pts) Find the derivative of $f(x) = 2^{\tan^3(x)}$

$$f'(x) = \left(2^{\tan^3 x}\right)\left(3\tan^2 x\right)\sec^2 x \ln 2$$

17. (7 pts) Find $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x$. Show all steps.

Form: 1^∞

$$y = \left(1-\frac{2}{x}\right)^x$$

$$\ln y = x\ln\left(1-\frac{2}{x}\right)$$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\ln\left(1-\frac{2}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\ln\left(1-\frac{2}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0}$$

$$\overset{L}{=} \lim_{x\to\infty}\frac{\frac{\frac{2}{x^2}}{1-\frac{2}{x}}}{\frac{-1}{x^2}} = \lim_{x\to\infty}\frac{-2}{1-\frac{2}{x}}$$

$$= -2.$$

$$\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x = e^{-2}$$

18. Consider $f(x) = x\ln(x) + 2x$

a.) (2 pts) Find the domain of $f(x)$.

$x > 0$

b.) (4 pts) Find $\lim\limits_{x\to 0^+} f(x)$.

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \quad \frac{-\infty}{\infty}$$

$$\overset{L}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} (-x)$$

$$= \boxed{0}$$

so $\lim\limits_{x\to 0^+} (x\ln x + 2x) = 0$

c.) (6 pts) Find the interval(s) where $f(x)$ is increasing or decreasing and identify the local extrema of $f(x)$.

$f'(x) = \ln x + x \cdot \frac{1}{x} + 2$

$f'(x) = \ln x + 3$

$f'(x) = 0$

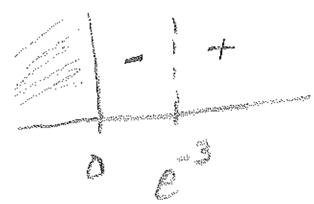
$\ln x + 3 = 0$

$\ln x = -3 \rightarrow x = e^{-3}$

$f(e^{-3}) = e^{-3}(-3) + 2e^{-3}$

$= -3e^{-3} + 2e^{-3}$

$= -e^{-3}$



dec: $(0, e^{-3})$

inc: (e^{-3}, ∞)

min: $(e^{-3}, -e^{-3})$

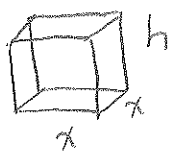
d.) (4 pts) Find the interval(s) of concavity of $f(x)$.

$f''(x) = \frac{1}{x}$

$f''(x) > 0$ for $x > 0$

$f(x)$ is concave up on $(0, \infty)$

19. (8 pts) A closed box with a square base is to be constructed. If the surface area of the box is 16 square centimeters, find the dimensions of the box with maximum volume.



$A = 16$

$2x^2 + 4xh = 16 \rightarrow h = \frac{16-2x^2}{4x}$

maximize $V = x^2 h$

$V = x^2\left(\frac{16-2x^2}{4x}\right)$

$V = \frac{1}{4}x(16-2x^2)$

$V = 4x - \frac{1}{2}x^3$

$V' = 4 - \frac{3}{2}x^2$

$V' = 0$

$4 - \frac{3}{2}x^2 = 0$

$\frac{3}{2}x^2 = 4$

$x^2 = \frac{8}{3}$

$x = \sqrt{\frac{8}{3}}$

$\nearrow \quad \searrow$

$+ \quad -$

$1 \quad \sqrt{\frac{8}{3}} \quad 3$

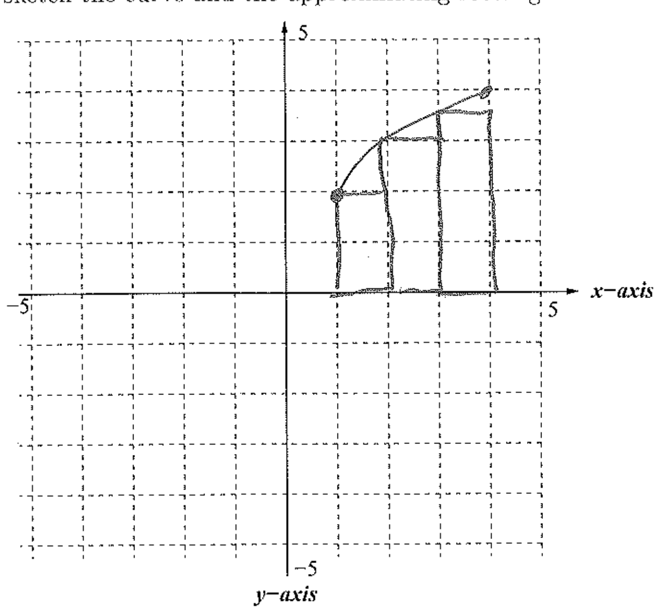
$h = \frac{16 - \frac{16}{3}}{4\sqrt{8/3}} = \frac{32/3}{4\sqrt{8/3}} = \frac{8}{3\sqrt{8/3}}$

dimensions

$x = \sqrt{\frac{8}{3}}$ cm

$h = \frac{8}{3\sqrt{8/3}}$ cm

20. (6 pts) Approximate the area under the graph of $f(x) = 2\sqrt{x}$, above the x-axis, from $x = 1$ to $x = 4$ using the partition points $P = \{1, 2, 3, 4\}$. Choose x_i^* to be the left endpoint of the ith subinterval. Use the grid below to sketch the curve and the approximating rectangles.



$A \approx \left(f(1) + f(2) + f(3)\right)(1)$

$\approx 2 + 2\sqrt{2} + 2\sqrt{3}$

21. (8 pts) Suppose it is known that $\mathbf{r}'(\mathbf{t}) = \langle \cos t + t, e^t + 1 \rangle$ and $\mathbf{r}(0) = \langle -1, 1 \rangle$. Find $\mathbf{r}(\mathbf{t})$.

$$r(t) = \langle \sin t + \frac{t^2}{2} + C_1, e^t + t + C_2 \rangle$$

$$r(0) = \langle C_1, 1 + C_2 \rangle$$

$$C_1 = -1$$

$$1 + C_2 = 1$$

$$C_2 = 0$$

$$r(t) = \langle \sin t + \frac{t^2}{2} - 1, e^t + t \rangle$$

22. (5 pts) The number of bacteria in a culture is increasing according to the law of exponential growth. If there are initially 125 bacteria and the number of bacteria at time $t = 2$ hours is 350 bacteria, find the number of bacteria, $y(t)$, in the culture at time t hours.

$$y(t) = 125e^{kt}$$

$$y(2) = 350$$

$$350 = 125e^{2k}$$

$$\frac{14}{5} = e^{2k}$$

$$\ln \frac{14}{5} = 2k$$

$$k = \frac{1}{2} \ln \frac{14}{5}$$

$$y(t) = 125e^{\frac{t}{2} \ln \frac{14}{5}}$$

$$\boxed{y(t) = 125\left(\frac{14}{5}\right)^{t/2}}$$