

MATH 151, FALL 2013  
COMMON EXAM III - VERSION B

LAST NAME: Key FIRST NAME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

UIN: \_\_\_\_\_

**DIRECTIONS:**

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-21), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

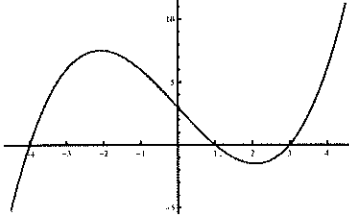
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**DO NOT WRITE BELOW!**

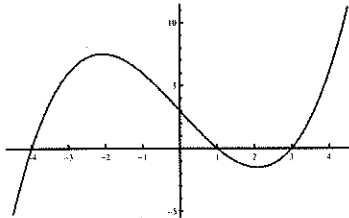
Question	Points Awarded	Points
1-15		45
16		5
17		7
18		16
19		8
20		6
21		8
22		5
		100

PART I: Multiple Choice. 3 points each

1. If the graph below is that of the derivative,  $f'(x)$ , of a function  $f(x)$ , where is  $f(x)$  increasing?



- (a)  $(-4, 1) \cup (3, \infty)$   
 (b)  $(-\infty, -4) \cup (1, 3)$   
 (c)  $(-2, 2)$   
 (d)  $(-\infty, -2) \cup (2, \infty)$   
 (e)  $(-\infty, 0)$
2. If the graph below is that of the derivative,  $f'(x)$ , of a function  $f(x)$ , what are the  $x$  coordinates of the inflection point(s) of  $f(x)$ ?



- (a)  $x = 0$   
 (b)  $x = \pm 2$   
 (c)  $x = 1$   
 (d)  $x = -4, x = 1, x = 3$   
 (e) There are no points of inflection

3. Find  $\frac{dy}{dx}$  if  $y = (1 + 2x)^{x^2}$ .

- (a)  $\frac{dy}{dx} = (1 + 2x)^{x^2} \left( 2x \ln(1 + 2x) + \frac{x^2}{1 + 2x} \right)$   
 (b)  $\frac{dy}{dx} = \left( 2x \ln(1 + 2x) + \frac{2x^2}{1 + 2x} \right)$   
 (c)  $\frac{dy}{dx} = (1 + 2x)^{x^2} \left( 2x \ln(1 + 2x) + \frac{2x^2}{1 + 2x} \right)$   
 (d)  $\frac{dy}{dx} = (1 + 2x)^{x^2} \left( \frac{4x}{1 + 2x} \right)$   
 (e)  $\frac{dy}{dx} = x^2(1 + 2x)^{x^2-1}(2)$

$$y = (1 + 2x)^{x^2}$$

$$\ln y = \ln(1 + 2x)^{x^2}$$

$$\ln y = x^2 \ln(1 + 2x)$$

$$\frac{1}{y} y' = 2x \ln(1 + 2x) + x^2 \frac{2}{1 + 2x}$$

$$y' = y \left( 2x \ln(1 + 2x) + \frac{2x^2}{1 + 2x} \right)$$

$$2 y' = (1 + 2x)^{x^2} \left[ 2x \ln(1 + 2x) + \frac{2x^2}{1 + 2x} \right]$$

4. Find  $f'(e^2)$  if  $f(x) = \ln(1 + \ln x)$ .

(a)  $\frac{1}{2e^2}$

(b)  $\frac{e^2}{1+e^2}$

(c)  $\frac{1}{1+e^2}$

(d)  $\frac{1}{e^2 + 2e}$

(e)  $\frac{1}{3e^2}$

$$f'(x) = \frac{\frac{1}{x}}{1 + \ln x}$$

$$f'(e^2) = \frac{\frac{1}{e^2}}{1 + \ln e^2} = \frac{\frac{1}{e^2}}{3} = \frac{1}{3e^2}$$

5. Calculate  $\cos(\arctan(2))$ .

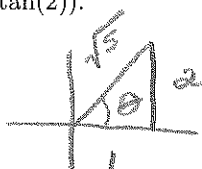
(a) 2

(b)  $\frac{\sqrt{3}}{2}$

(c)  $\frac{2}{\sqrt{5}}$

(d)  $\frac{2}{\sqrt{3}}$

(e)  $\frac{1}{\sqrt{5}}$



$$\cos \theta = \frac{1}{\sqrt{5}}$$

6. Find  $f''(1)$  if  $f(x) = \arctan(2x)$ .

(a)  $-\frac{8}{9}$

(b)  $-\frac{16}{25}$

(c)  $-\frac{4}{9}$

(d)  $-\frac{8}{25}$

(e)  $-\frac{2}{25}$

$$f'(x) = \frac{2}{1+4x^2}$$

$$f''(x) = \frac{-16x}{(1+4x^2)^2}$$

$$f''(1) = \frac{-16}{25}$$

7. Given that  $f(x)$  is defined everywhere except  $x = -3$  and  $f'(x) = \frac{x^2(1-x)}{(x+3)^3}$ , on what intervals is  $f(x)$  increasing?

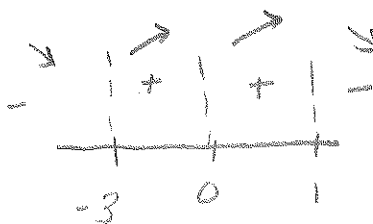
(a)  $(-3, 0) \cup (0, 1)$

(b)  $(-3, 0) \cup (1, \infty)$

(c)  $(-\infty, -3) \cup (1, \infty)$

(d)  $(-\infty, 0) \cup (0, 1)$

(e)  $(-\infty, 0) \cup (1, \infty)$



8. Find the absolute maximum and absolute minimum values for  $f(x) = (x^2 - 1)^3$  on the interval  $[-2, 2]$ .

- (a) Absolute maximum: 27. Absolute minimum: 0.
- (b) Absolute maximum: 125. Absolute minimum: -1.
- (c) Absolute maximum: 108. Absolute minimum: -108.
- (d) Absolute maximum: 27. Absolute minimum: -1.
- (e) Absolute maximum: 125. Absolute minimum: 0.

$$f'(x) = 3(x^2 - 1)^2 (2x)$$

$$\text{CR: } x = 0, \pm 1$$

$$f(0) = -1 \text{ min}$$

$$f(\pm 1) = 0$$

$$f(\pm 2) = 27 \text{ max}$$

9. Find an antiderivative of  $f(x) = \frac{x^3 - x^2 + 5}{x^4}$ .

- (a)  $\ln|x| + \frac{1}{x} - \frac{1}{x^5} + C$
- (b)  $\ln|x| + \frac{1}{x} - \frac{5}{3x^3} + C$
- (c)  $\ln|x| + \frac{1}{3x^3} - \frac{5}{3x^3} + C$
- (d)  $\ln|x| + \frac{1}{3x^3} - \frac{1}{x^5} + C$
- (e)  $\frac{x^0}{0} + \frac{1}{x} - \frac{5}{3x^3} + C$

$$f(x) = \frac{1}{x} - \frac{1}{x^2} + \frac{5}{x^4}$$

$$F(x) = \ln|x| + \frac{1}{x} - \frac{5}{3x^3} + C$$

10. Find  $\lim_{x \rightarrow 1} \frac{e^{2x-2} + x^2 - 2}{\ln(x) + 2x - 2}$ .

- (a) 0
- (b) 1
- (c) 2
- (d)  $\frac{2}{3}$
- (e)  $\frac{4}{3}$

$$\frac{0}{0} \stackrel{L}{=} \lim_{x \rightarrow 1}$$

$$\frac{e^{2x-2} + 2x}{\frac{1}{x} + 2} = \frac{4}{3}$$

11.  $\lim_{x \rightarrow \infty} (\ln(x+2) - \ln(3x+1)) = \lim_{x \rightarrow \infty} \ln\left(\frac{x+2}{3x+1}\right)$

- (a) 0
- (b)  $\ln \frac{1}{3}$
- (c) 1
- (d)  $-\infty$
- (e)  $\infty$

$$= \ln \frac{1}{3}$$

12. Find the sum  $\sum_{i=2}^{98} \left( \frac{1}{i+1} - \frac{1}{i+2} \right) = \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{99} - \frac{1}{100}$

- (a)  $\frac{1}{3}$
- (b) 0
- (c)  $\frac{97}{300}$
- (d)  $\frac{1}{100}$
- (e)  $\frac{49}{300}$

$$= \frac{1}{3} - \frac{1}{100}$$

$$= \frac{97}{300}$$

13. Find the inflection point(s) for  $f(x) = xe^{2x}$ .

- (a)  $(-\frac{1}{2}, -\frac{1}{2}e^{-1})$
- (b)  $(-1, -e^{-2})$
- (c) (0, 0)
- (d)  $(-\frac{1}{4}, -\frac{1}{4}e^{-1/2})$
- (e)  $(-2, -2e^{-4})$

$$f'(x) = e^{2x} + 2xe^{2x}$$

$$f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x}$$

$$= 4e^{2x} + 4xe^{2x}$$

$$= 4e^{2x}(1+x)$$

IP:  $(-1, -1e^{-2})$

14. Find the x coordinate where  $f(x) = x\sqrt{x+1}$  has a local minimum.

- (a)  $x = -\frac{2}{3}$
- (b)  $x = -\frac{1}{3}$
- (c)  $x = -1$
- (d)  $x = 0$
- (e) None,  $f(x)$  is always increasing.

$$f'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

$$= \frac{2(x+1) + x}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}}$$

15. Solve  $\log_2(x^2 - 38) - \log_2(5 - x) = 1$  for x.

- (a)  $x = 6$
- (b)  $x = -8, x = 6$
- (c)  $x = 8$
- (d)  $x = -8$
- (e) No solution

$$\log_2 \frac{x^2 - 38}{5 - x} = 1$$

$$\frac{x^2 - 38}{5 - x} = 2$$

$$x^2 - 38 = 2(5 - x)$$

$$x^2 - 38 = 10 - 2x$$

$$x^2 + 2x - 48 = 0$$

$$(x+8)(x-6) = 0$$

$x = -8$

PART II: Work Out

**Directions:** Present your solutions in the space provided. Show all your work neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (5 pts) Find the derivative of  $f(x) = 7^{\sec^3(x)}$

$$f'(x) = \left( 7^{\sec^3 x} \right) (3 \sec^2 x) (\sec x \tan x) \ln 7$$

17. (7 pts) Find  $\lim_{x \rightarrow \infty} \left( 1 - \frac{4}{x} \right)^x$ . Show all steps.

Form  $1^\infty$

$$y = \left( 1 - \frac{4}{x} \right)^x$$

$$\ln y = \ln \left( 1 - \frac{4}{x} \right)^x$$

$$= x \ln \left( 1 - \frac{4}{x} \right)$$

$$\lim_{x \rightarrow \infty} x \ln \left( 1 - \frac{4}{x} \right)$$

$$\lim_{x \rightarrow \infty} \frac{\ln \left( 1 - \frac{4}{x} \right)}{\frac{1}{x}} \quad \frac{0}{0}$$

$$\stackrel{L}{=} \lim_{x \rightarrow \infty} \frac{\frac{4}{x^2}}{1 - \frac{4}{x}} \cdot \frac{1}{-\frac{1}{x^2}}$$

$$\lim_{x \rightarrow \infty} \frac{-4}{1 - \frac{4}{x}} = -4$$

$$\lim_{x \rightarrow \infty} \left( 1 - \frac{4}{x} \right)^x = e^{-4}$$

18. Consider  $f(x) = x \ln(x) + 3x$

a.) (2 pts) Find the domain of  $f(x)$ .

$$x > 0$$

b.) (4 pts) Find  $\lim_{x \rightarrow 0^+} f(x)$ .

$$\lim_{x \rightarrow 0^+} (x \ln x + 3x) = \lim_{x \rightarrow 0^+} x \ln x + \lim_{x \rightarrow 0^+} 3x \rightarrow 0$$

$$\lim_{x \rightarrow 0^+} x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} = \frac{-\infty}{\infty}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} (-x) = 0$$

c.) (6 pts) Find the interval(s) where  $f(x)$  is increasing or decreasing and identify the local extrema of  $f(x)$ .

$$f'(x) = \ln x + x \cdot \frac{1}{x} + 3$$

$$f'(x) = \ln x + 4$$

$$f'(x) = 0$$

$$\ln x + 4 = 0$$

$$\ln x = -4$$

$$x = e^{-4}$$



f dec  $(-\infty, e^{-4})$   
f inc  $(e^{-4}, \infty)$

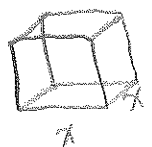
Local min  
 $(e^{-4}, f(e^{-4}))$   
 $= (e^{-4}, -e^{-4})$

d.) (4 pts) Find the interval(s) of concavity of  $f(x)$ .

$$f''(x) = \frac{1}{x} > 0 \text{ on } (0, \infty)$$

f concave up on  $(0, \infty)$

19. (8 pts) A closed box with a square base is to be constructed. If the surface area of the box is 20 square centimeters, find the dimensions of the box with maximum volume.



$$A = 2x^2 + 4xh = 20 \rightarrow h = \frac{20 - 2x^2}{4x}$$

$$V = x^2 h$$

$$= x^2 \left( \frac{20 - 2x^2}{4x} \right)$$

$$= \frac{1}{4} x (20 - 2x^2)$$

$$V = 5x - \frac{1}{2} x^3$$

$$V' = 5 - \frac{3}{2} x^2$$

$$V' = 0$$

$$5 - \frac{3}{2} x^2 = 0$$

$$\frac{3}{2} x^2 = 5$$

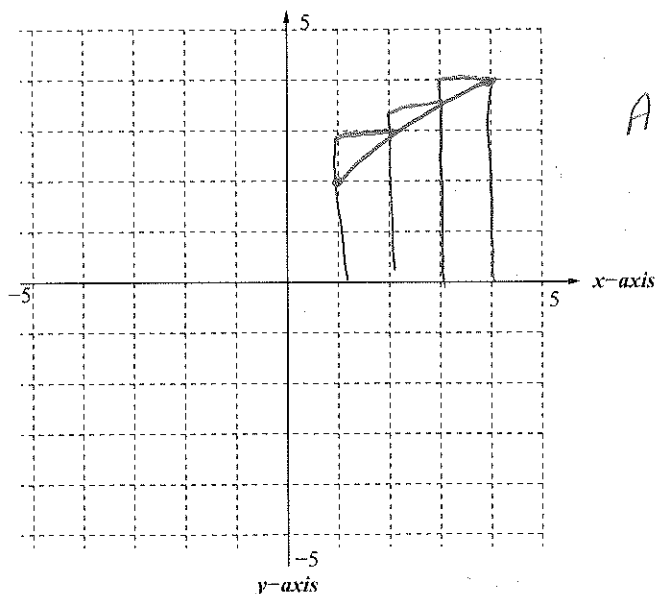
$$x^2 = \frac{10}{3}$$

$$x = \sqrt{\frac{10}{3}} \text{ cm}$$

$$h = \frac{20 - \frac{20}{3}}{4 \sqrt{\frac{10}{3}}} \text{ cm}$$



20. (6 pts) Approximate the area under the graph of  $f(x) = 2\sqrt{x}$ , above the  $x$ -axis, from  $x = 1$  to  $x = 4$  using the partition points  $P = \{1, 2, 3, 4\}$ . Choose  $x_i^*$  to be the right endpoint of the  $i$ th subinterval. Use the grid below to sketch the curve and the approximating rectangles.



$$A \approx (f(2) + f(3) + f(4))(1)$$

$$A \approx 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{4}$$



21. (8 pts) Suppose it is known that  $r'(t) = \langle e^t + 1, \cos t \rangle$  and  $r(0) = \langle -1, 1 \rangle$ . Find  $r(t)$ .

$$r(t) = \langle e^t + t + C_1, \sin t + C_2 \rangle$$

$$r(0) = \langle 1 + C_1, C_2 \rangle$$

$$1 + C_1 = -1 \quad C_2 = 1$$

$$C_1 = -2$$

$$r(t) = \langle e^t + t - 2, \sin t + 1 \rangle$$

22. (5 pts) The number of bacteria in a culture is increasing according to the law of exponential growth. If there are initially 120 bacteria and the number of bacteria at time  $t = 3$  hours is 320 bacteria, find the number of bacteria,  $y(t)$ , in the culture at time  $t$  hours.

$$y(t) = 120 e^{kt}$$

$$y(3) = 320$$

$$320 = 120 e^{3k}$$

$$\frac{8}{3} = e^{3k}$$

$$\ln \frac{8}{3} = 3k$$

$$k = \frac{1}{3} \ln \frac{8}{3}$$

$$\frac{t}{3} \ln \frac{8}{3}$$

$$y(t) = 120 e^{\frac{t}{3} \ln \frac{8}{3}}$$

$$y(t) = 120 \left( \frac{8}{3} \right)^{t/3}$$

