Student (Print) ________________________________ Section __________

Last, First Middle

Student (Sign) ________________________________

Student ID ________________________________

Instructor ________________________________

# MATH 152
# Exam1
# Fall 2000
# Test Form A
# Solutions

| | |
|---|---|
| 1-10 | /50 |
| 11 | /10 |
| 12 | /10 |
| 13 | /10 |
| 14 | /10 |
| 15 | /10 |
| TOTAL | |

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Formulas:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\cos^2 A = \frac{1+\cos 2A}{2}$$

$$\sin A\sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

$$\sin A\cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

$$\cos A\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

$$\int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\,d\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

**1.** What is the average value of the function $f(x) = x^3$ on the interval $[-1, 2]$.

**a.** $\frac{5}{4}$ correctchoice
**b.** $\frac{15}{4}$
**c.** $4$
**d.** $\frac{4}{3}$
**e.** $\frac{20}{3}$

$$f_{ave} = \frac{1}{3}\int_{-1}^{2} x^3\,dx = \frac{1}{3}\left[\frac{x^4}{4}\right]_{-1}^{2} = \frac{1}{12}[16-1] = \frac{15}{12} = \frac{5}{4}$$

**2.** Using a trigonometric substitution, the integral $\int \frac{dx}{\sqrt{9+x^2}}$ becomes:

**a.** $\int \sec\theta\,d\theta$ correctchoice
**b.** $\int \frac{d\theta}{3\sec\theta}$
**c.** $\int \frac{d\theta}{\sec\theta}$
**d.** $\int 3\tan\theta\,d\theta$
**e.** $\int \frac{d\theta}{\tan\theta}$

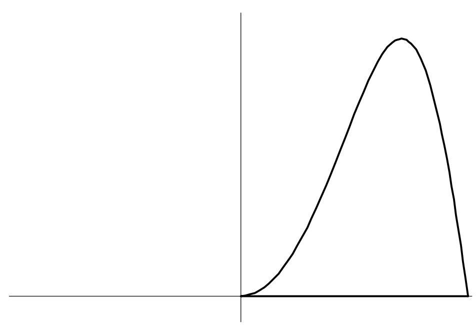
$x = 3\tan\theta \qquad dx = 3\sec^2\theta\,d\theta \qquad \sqrt{9+x^2} = \sqrt{9+9\tan^2\theta} = 3\sec\theta$

$$\int \frac{dx}{\sqrt{9+x^2}} = \int \frac{3\sec^2\theta\,d\theta}{3\sec\theta} = \int \sec\theta\,d\theta$$

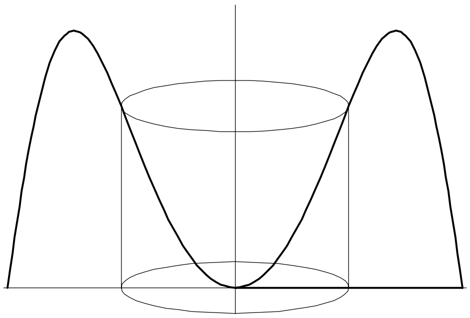
**3.** The region bounded by the curves

$y = \sin(x^2)$, $y = 0$, $x = 0$ and $x = \sqrt{\pi}$

is rotated about the $y$-axis. Find the volume.



**a.** $1$
**b.** $2$
**c.** $\pi$
**d.** $2\pi$ correctchoice
**e.** $4\pi$



$x$-integral, cylinders, radius $r = x$,
height $h = \sin(x^2)$

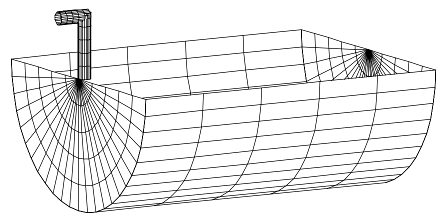
$$V = \int 2\pi r\, h\, dx = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\, dx$$

Let $u = x^2$ $\quad du = 2x\, dx$.

$$V = \int_0^{\pi} \pi \sin(u)\, du = \pi\Big[-\cos u\Big]_0^{\pi}$$

$$= \pi(--1 - -1) = 2\pi$$

**4**. A tank has the shape of a half cylinder which is $5$ m long and $2$ m in radius laying on its side. The tank is full of water. Which integral gives the work done to pump the water out of a spout which is $1$ m above the tank.
The density of water is $\rho = 1000$ kg/m$^3$.
The acceleration of gravity is $g = 9.8$ m/sec$^2$.
Measure $y$ down from the axis of the cylinder.



**a**. $9800 \int_{-2}^{2} (y+1)10\sqrt{4-y^2}\, dy$

**b**. $9800 \int_{-2}^{2} (1-y)5\sqrt{4-y^2}\, dy$

**c**. $9800 \int_{0}^{2} (1-y)10(2-y)\, dy$

**d**. $9800 \int_{0}^{2} (y+1)5(2-y)\, dy$

**e**. $9800 \int_{0}^{2} (y+1)10\sqrt{4-y^2}\, dy$ correctchoice

A slice of the water at depth $y$ and thickness $\Delta y$ must be lifted a distance $D = y + 1$. Its length is $5$ and its width is $2\sqrt{4-y^2}$. So its volume is $\Delta V = 10\sqrt{4-y^2}\,\Delta y$ and its weight is $\Delta F = \rho g \Delta V$. So the work to lift the slice is $\Delta W = D\Delta F$. We add up the work for the slices between $y = 0$ and $y = 2$. So the total work is

$$W = 9800 \int_0^2 (y+1)10\sqrt{4-y^2}\, dy$$

**5**. Compute $\int_0^1 (x-2)\, e^x\, dx$.

**a**. $1 - 2e$

**b**. $1$

**c**. $3 - 2e$ correctchoice

**d**. $-2e$

**e**. $3$

$u = x - 2 \qquad dv = e^x\, dx$

$du = dx \qquad v = e^x$

$$\int_0^1 (x-2)\, e^x\, dx = \left[ (x-2)\, e^x - \int_0^1 e^x\, dx \right]_0^1 = \Big[ (x-2)\, e^x - e^x \Big]_0^1 = [(1-2)\, e - e] - [(-2) - 1] = 3 - 2e$$

**6.** Evaluate $\int \cos^3 x\,dx$.

**a.** $\dfrac{\sin^4 x}{4} + C$

**b.** $\dfrac{\sin^2 x}{2} - \dfrac{\sin^4 x}{4} + C$

**c.** $\dfrac{\cos^4 x}{4} + C$

**d.** $\sin x - \dfrac{\sin^3 x}{3} + C$ correctchoice

**e.** $-\dfrac{\cos^4 x}{4} + C$

$u = \sin x \qquad du = \cos x\,dx \qquad \cos^2 x = 1 - \sin^2 x = 1 - u^2$

$$\int \cos^3 x\,dx = \int \cos^2 x \cos x\,dx = \int (1 - u^2)\,du = u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$$

**7.** Which of these integrals represents the area between the curves $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \pi$.

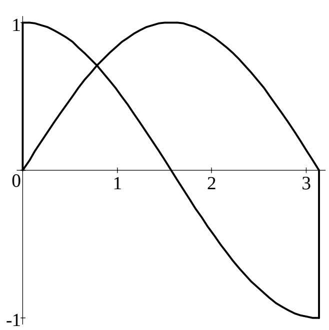
**a.** $\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{3\pi/4} (\sin x - \cos x)\,dx + \int_{3\pi/4}^{\pi} (\cos x - \sin x)\,dx$

**b.** $\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\,dx$ correctchoice

**c.** $\int_0^{\pi/4} (\sin x - \cos x)\,dx + \int_{\pi/4}^{\pi} (\cos x - \sin x)\,dx$

**d.** $\int_0^{\pi} (\cos x - \sin x)\,dx$

**e.** $\int_0^{\pi} (\sin x - \cos x)\,dx$




On $0 < x < \dfrac{\pi}{4}$, $\cos x > \sin x$.

On $\dfrac{\pi}{4} < x < \pi$, $\cos x < \sin x$.

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\,dx$$

**8.** Evaluate $\int_0^4 x\sqrt{9 + x^2}\,dx$.

**a.** $\dfrac{147}{2}$

**b.** $\dfrac{98}{3}$ correctchoice

**c.** $6$

**d.** $\dfrac{392}{3}$

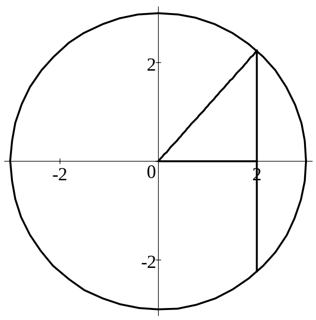
**e.** $\dfrac{8}{3}$

$u = 9 + x^2 \qquad du = 2x\,dx \qquad \dfrac{1}{2}\,du = x\,dx$

$$\int_0^4 x\sqrt{9 + x^2}\,dx = \frac{1}{2}\int_9^{25} u^{1/2}\,du = \frac{1}{2}\,\frac{2u^{3/2}}{3}\Big|_9^{25} = \frac{1}{3}(125 - 27) = \frac{98}{3}$$

**9.** The base of a solid is the circle $x^2 + y^2 = 9$. The cross sections perpendicular to the $x$-axis are squares. Find the volume.

**a.** $9\pi$
**b.** $36$
**c.** $72$
**d.** $81\pi$
**e.** $144$ correctchoice




$x$-integral

Side of square $= s = 2y = 2\sqrt{9 - x^2}$

Area of square $= A(x) = s^2 = 4(9 - x^2)$

Volume $= V = \int A(x)\,dx = \int_{-3}^{3} 4(9 - x^2)\,dx$

$$= 4\left[9x - \frac{x^3}{3}\right]_{-3}^{3} = 4[18] - 4[-18] = 144$$

**10.** What is the form of the partial fraction decomposition of $\dfrac{x^2+3}{x^3 - 2x^2 + x}$?

**a.** $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-1)^2}$
**b.** $\dfrac{A}{x^3} + \dfrac{B}{2x^2} + \dfrac{C}{x}$
**c.** $\dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$ correctchoice
**d.** $\dfrac{A}{x} + \dfrac{B}{(x-1)^2}$
**e.** $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2}$

$$\frac{x^2+3}{x^3 - 2x^2 + x} = \frac{x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

You may not use a calculator.

**11.** Compute $\int x^3 \ln x \, dx$.

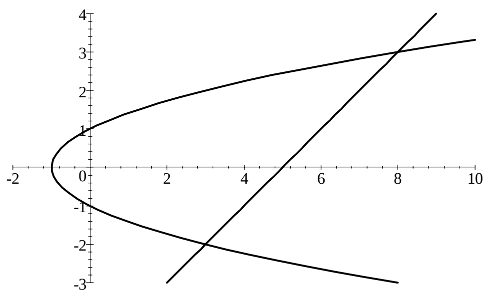
$u = \ln x \qquad dv = x^3\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^4}{4}$

$$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\frac{1}{x}\,dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

**12.** Find the area between the curves $x = y^2 - 1$ and $y = x - 5$.

**a.** (4 pts) Graph the curves.




**b.** (4 pts) Set up the integral(s) for the area.

$y^2 - 1 = y + 5 \qquad y^2 - y - 6 = 0 \qquad y = -2, 3$

$$A = \int_{-2}^{3} ((y+5) - (y^2 - 1))\,dy$$

**c.** (2 pts) Compute the area.

$$A = \int_{-2}^{3} (-y^2 + y + 6)\,dy = \left[-\frac{y^3}{3} + \frac{y^2}{2} + 6y\right]_{-2}^{3} = \left[-9 + \frac{9}{2} + 18\right] - \left[\frac{8}{3} + 2 - 12\right] = \frac{125}{6}$$

**13.** Compute $\int_0^{\pi/3} \tan^3 x \sec x \, dx$.

$u = \sec x \qquad du = \sec x \tan x \, dx \qquad \tan^2 x = \sec^2 x - 1 = u^2 - 1$

$$\int_0^{\pi/3} \tan^3 x \sec x\,dx = \int_0^{\pi/3} \tan^2 x \sec x \tan x\,dx = \int_1^2 (u^2 - 1)\,du = \left[\frac{u^3}{3} - u\right]_1^2 = \left[\frac{8}{3} - 2\right] - \left[\frac{1}{3} - 1\right] = \frac{4}{3}$$

**14.** Evaluate $\displaystyle\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$.

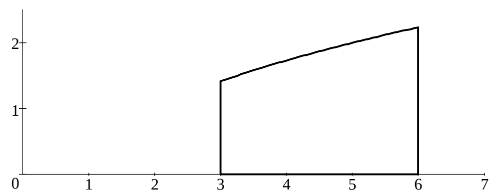
$$x = \sin\theta \qquad dx = \cos\theta\,d\theta \qquad \sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$$

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/4} \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta = \int_0^{\pi/4} \sin^2\theta\,d\theta = \int_0^{\pi/4} \frac{1-\cos 2\theta}{2}\,d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4} = \frac{1}{2}\left[\frac{\pi}{4} - \frac{\sin\frac{\pi}{2}}{2}\right] - 0 = \frac{\pi}{8} - \frac{1}{4}$$

**15.** Consider the region in the plane bounded by the curves $\sqrt{x-1}$, $x = 3$, $x = 6$ and $y = 0$.

**a.** (3 pts) Graph the region.




**b.** (1 pt) The region is rotated about the $x$-axis. To find the volume, you will use

an $x$-integral a $y$-integral (Circle one.)

with

disks washers cylindrical shells (Circle one.)

Correct: $x$-integral with disks OR $y$-integral with cylindrical shells

**c.** (4 pts) Set up the integral(s) for the volume.

$x$-integral: $\displaystyle V = \int_3^6 \pi R^2\,dx = \int_3^6 \pi\left(\sqrt{x-1}\right)^2 dx$

$y$-integral: $\displaystyle V = \int_0^{\sqrt{2}} 2\pi rh\,dy + \int_{\sqrt{2}}^{\sqrt{5}} 2\pi rh\,dy = \int_0^{\sqrt{2}} 2\pi y(6-3)\,dy + \int_{\sqrt{2}}^{\sqrt{5}} 2\pi y(6-(1+y^2))\,dy$

**d.** (2 pts) Compute the volume.

$x$-integral: $\displaystyle V = \pi\int_3^6 (x-1)\,dx = \pi\left[\frac{x^2}{2} - x\right]_3^6 = \pi\left[\frac{36}{2} - 6\right] - \pi\left[\frac{9}{2} - 3\right] = \frac{21}{2}\pi$

$y$-integral: $\displaystyle V = \int_0^{\sqrt{2}} 6\pi y\,dy + \int_{\sqrt{2}}^{\sqrt{5}} 2\pi y(5-y^2)\,dy = \left[3\pi y^2\right]_0^{\sqrt{2}} + \left[5\pi y^2 - \pi\frac{y^4}{2}\right]_{\sqrt{2}}^{\sqrt{5}}$

$$= 6\pi + \left[25\pi - \frac{25}{2}\pi\right] - [10\pi - 2\pi] = \frac{21}{2}\pi$$