

Student (Print) \_\_\_\_\_  
 Last, First Middle

Student (Sign) \_\_\_\_\_

Student ID \_\_\_\_\_

Instructor \_\_\_\_\_

Section \_\_\_\_\_

MATH 152  
 Exam1  
 Fall 2000  
 Test Form B  
 Solutions

1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Formulas:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin A \sin B = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$$

$$\sin A \cos B = \frac{1}{2} \sin(A - B) + \frac{1}{2} \sin(A + B)$$

$$\cos A \cos B = \frac{1}{2} \cos(A - B) + \frac{1}{2} \cos(A + B)$$

$$\int \sec \theta d\theta = \ln|\sec \theta + \tan \theta| + C$$

$$\int \csc \theta d\theta = \ln|\csc \theta - \cot \theta| + C$$

$$\int \ln x dx = x \ln x - x + C$$

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Evaluate  $\int_0^4 x\sqrt{9+x^2} dx$ .

- a.  $\frac{147}{2}$
- b.  $\frac{98}{3}$  correctchoice
- c. 6
- d.  $\frac{392}{3}$
- e.  $\frac{8}{3}$

$$u = 9 + x^2 \quad du = 2x dx \quad \frac{1}{2} du = x dx$$

$$\int_0^4 x\sqrt{9+x^2} dx = \frac{1}{2} \int_9^{25} u^{1/2} du = \frac{1}{2} \frac{2u^{3/2}}{3} \Big|_9^{25} = \frac{1}{3} (125 - 27) = \frac{98}{3}$$

2. Evaluate  $\int \cos^3 x dx$ .

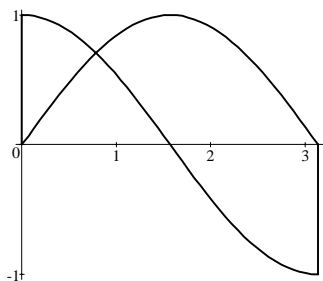
- a.  $\frac{\sin^4 x}{4} + C$
- b.  $\frac{\sin^2 x}{2} - \frac{\sin^4 x}{4} + C$
- c.  $\frac{\cos^4 x}{4} + C$
- d.  $\sin x - \frac{\sin^3 x}{3} + C$  correctchoice
- e.  $-\frac{\cos^4 x}{4} + C$

$$u = \sin x \quad du = \cos x dx \quad \cos^2 x = 1 - \sin^2 x = 1 - u^2$$

$$\int \cos^3 x dx = \int \cos^2 x \cos x dx = \int (1 - u^2) du = u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$$

3. Which of these integrals represents the area between the curves  $y = \sin x$  and  $y = \cos x$  from  $x = 0$  to  $x = \pi$ .

- a.  $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{3\pi/4} (\sin x - \cos x) dx + \int_{3\pi/4}^{\pi} (\cos x - \sin x) dx$
- b.  $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$  correctchoice
- c.  $\int_0^{\pi/4} (\sin x - \cos x) dx + \int_{\pi/4}^{\pi} (\cos x - \sin x) dx$
- d.  $\int_0^{\pi} (\cos x - \sin x) dx$
- e.  $\int_0^{\pi} (\sin x - \cos x) dx$



On  $0 < x < \frac{\pi}{4}$ ,  $\cos x > \sin x$ .

On  $\frac{\pi}{4} < x < \pi$ ,  $\cos x < \sin x$ .

$$A = \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$$

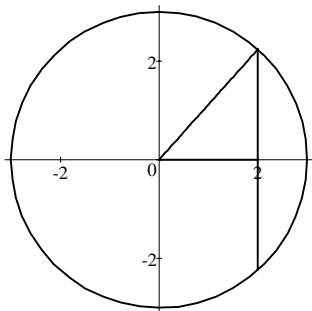
4. What is the form of the partial fraction decomposition of  $\frac{x^2+3}{x^3-2x^2+x}$ ?

- a.  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)^2}$
- b.  $\frac{A}{x^3} + \frac{B}{2x^2} + \frac{C}{x}$
- c.  $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$  correctchoice
- d.  $\frac{A}{x} + \frac{B}{(x-1)^2}$
- e.  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

$$\frac{x^2+3}{x^3-2x^2+x} = \frac{x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

5. The base of a solid is the circle  $x^2 + y^2 = 9$ . The cross sections perpendicular to the  $x$ -axis are squares. Find the volume.

- a.  $9\pi$
- b. 36
- c. 72
- d.  $81\pi$
- e. 144 correctchoice



$x$ -integral

$$\text{Side of square} = s = 2y = 2\sqrt{9-x^2}$$

$$\text{Area of square} = A(x) = s^2 = 4(9-x^2)$$

$$\text{Volume} = V = \int A(x) dx = \int_{-3}^3 4(9-x^2) dx$$

$$= 4 \left[ 9x - \frac{x^3}{3} \right]_{-3}^3 = 4[18] - 4[-18] = 144$$

6. Compute  $\int_0^1 (x-2)e^x dx$ .

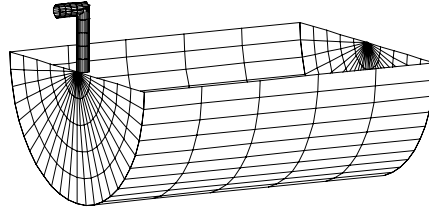
- a.  $1-2e$
- b. 1
- c.  $3-2e$  correctchoice
- d.  $-2e$
- e. 3

$$u = x-2 \quad dv = e^x dx$$

$$du = dx \quad v = e^x$$

$$\int_0^1 (x-2)e^x dx = \left[ (x-2)e^x - \int_0^1 e^x dx \right]_0^1 = \left[ (x-2)e^x - e^x \right]_0^1 = [(1-2)e - e] - [(-2) - 1] = 3 - 2e$$

7. A tank has the shape of a half cylinder which is 5 m long and 2 m in radius laying on its side. The tank is full of water. Which integral gives the work done to pump the water out of a spout which is 1 m above the tank. The density of water is  $\rho = 1000 \text{ kg/m}^3$ . The acceleration of gravity is  $g = 9.8 \text{ m/sec}^2$ . Measure  $y$  down from the axis of the cylinder.



- a.  $9800 \int_{-2}^2 (y + 1)10\sqrt{4 - y^2} dy$   
 b.  $9800 \int_{-2}^2 (1 - y)5\sqrt{4 - y^2} dy$   
 c.  $9800 \int_0^2 (1 - y)10(2 - y) dy$   
 d.  $9800 \int_0^2 (y + 1)5(2 - y) dy$   
 e.  $9800 \int_0^2 (y + 1)10\sqrt{4 - y^2} dy$      correctchoice

A slice of the water at depth  $y$  and thickness  $\Delta y$  must be lifted a distance  $D = y + 1$ . Its length is 5 and its width is  $2\sqrt{4 - y^2}$ . So its volume is  $\Delta V = 10\sqrt{4 - y^2} \Delta y$  and its weight is  $\Delta F = \rho g \Delta V$ . So the work to lift the slice is  $\Delta W = D \Delta F$ . We add up the work for the slices between  $y = 0$  and  $y = 2$ . So the total work is

$$W = 9800 \int_0^2 (y + 1)10\sqrt{4 - y^2} dy$$

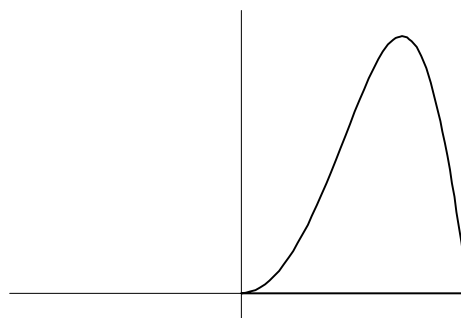
8. Using a trigonometric substitution, the integral  $\int \frac{dx}{\sqrt{9 + x^2}}$  becomes:

- a.  $\int \sec \theta d\theta$      correctchoice  
 b.  $\int \frac{d\theta}{3 \sec \theta}$   
 c.  $\int \frac{d\theta}{\sec \theta}$   
 d.  $\int 3 \tan \theta d\theta$   
 e.  $\int \frac{d\theta}{\tan \theta}$

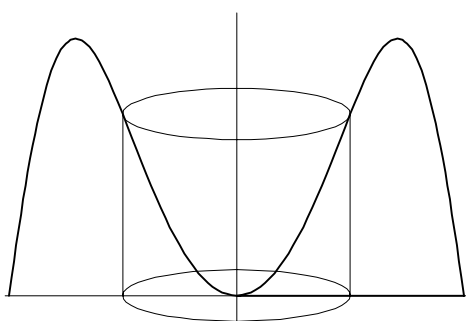
$$x = 3 \tan \theta \quad dx = 3 \sec^2 \theta d\theta \quad \sqrt{9 + x^2} = \sqrt{9 + 9 \tan^2 \theta} = 3 \sec \theta$$

$$\int \frac{dx}{\sqrt{9 + x^2}} = \int \frac{3 \sec^2 \theta d\theta}{3 \sec \theta} = \int \sec \theta d\theta$$

9. The region bounded by the curves  $y = \sin(x^2)$ ,  $y = 0$ ,  $x = 0$  and  $x = \sqrt{\pi}$  is rotated about the  $y$ -axis. Find the volume.



- a. 1
- b. 2
- c.  $\pi$
- d.  $2\pi$      correctchoice
- e.  $4\pi$



$x$ -integral, cylinders, radius  $r = x$ ,  
height  $h = \sin(x^2)$

$$V = \int 2\pi r h dx = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2) dx$$

Let  $u = x^2$       $du = 2x dx$ .

$$\begin{aligned} V &= \int_0^{\pi} \pi \sin(u) du = \pi [-\cos u]_0^{\pi} \\ &= \pi(-\cos \pi - (-\cos 0)) = \pi(-(-1) - (-1)) = 2\pi \end{aligned}$$

10. What is the average value of the function  $f(x) = x^3$  on the interval  $[-1, 2]$ .

- a.  $\frac{5}{4}$      correctchoice
- b.  $\frac{15}{4}$
- c. 4
- d.  $\frac{4}{3}$
- e.  $\frac{20}{3}$

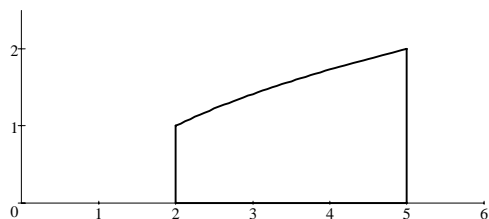
$$f_{ave} = \frac{1}{3} \int_{-1}^2 x^3 dx = \frac{1}{3} \left[ \frac{x^4}{4} \right]_{-1}^2 = \frac{1}{12} [16 - 1] = \frac{15}{12} = \frac{5}{4}$$

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

You may not use a calculator.

11. Consider the region in the plane bounded by the curves  $\sqrt{x-1}$ ,  $x = 2$ ,  $x = 5$  and  $y = 0$ .
- a. (3 pts) Graph the region.



- b. (1 pt) The region is rotated about the  $x$ -axis. To find the volume, you will use

an  $x$ -integral

a  $y$ -integral

(Circle one.)

with

disks

washers

cylindrical shells

(Circle one.)

Correct:  $x$ -integral with disks OR  $y$ -integral with cylindrical shells

- c. (4 pts) Set up the integral(s) for the volume.

$$x\text{-integral: } V = \int_2^5 \pi R^2 dx = \int_2^5 \pi (\sqrt{x-1})^2 dx$$

$$y\text{-integral: } V = \int_0^1 2\pi rh dy + \int_1^2 2\pi rh dy = \int_0^1 2\pi y(5-2) dy + \int_1^2 2\pi y(5-(1+y^2)) dy$$

- d. (2 pts) Compute the volume.

$$x\text{-integral: } V = \pi \int_2^5 (x-1) dx = \pi \left[ \frac{x^2}{2} - x \right]_2^5 = \pi \left[ \frac{25}{2} - 5 \right] - \pi \left[ \frac{4}{2} - 2 \right] = \frac{15}{2} \pi$$

$$y\text{-integral: } V = \int_0^1 6\pi y dy + \int_1^2 2\pi y(4-y^2) dy = \left[ 3\pi y^2 \right]_0^1 + \left[ 4\pi y^2 - \pi \frac{y^4}{2} \right]_1^2 \\ = 3\pi + [16\pi - 8\pi] - \left[ 4\pi - \frac{\pi}{2} \right] = \frac{15}{2} \pi$$

12. Evaluate  $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}} dx$ .

$$x = \sin \theta \quad dx = \cos \theta d\theta \quad \sqrt{1-x^2} = \sqrt{1-\sin^2 \theta} = \cos \theta$$

$$\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}} dx = \int_0^{\pi/6} \frac{\sin^2 \theta}{\cos \theta} \cos \theta d\theta = \int_0^{\pi/6} \sin^2 \theta d\theta = \int_0^{\pi/6} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{1}{2} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/6} = \frac{1}{2} \left[ \frac{\pi}{6} - \frac{\sin \frac{\pi}{3}}{2} \right] - 0 = \frac{\pi}{12} - \frac{\sqrt{3}}{8}$$

13. Compute  $\int_0^{\pi/4} \tan^3 x \sec x dx$ .

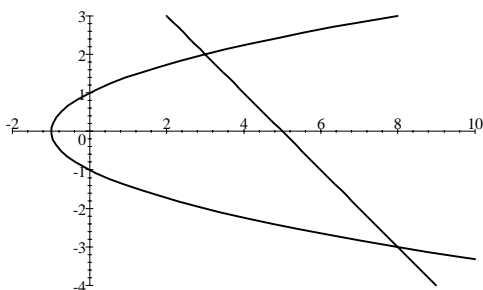
$$u = \sec x \quad du = \sec x \tan x dx \quad \tan^2 x = \sec^2 x - 1 = u^2 - 1$$

$$\int_0^{\pi/4} \tan^3 x \sec x dx = \int_0^{\pi/4} \tan^2 x \sec x \tan x dx = \int_1^{\sqrt{2}} (u^2 - 1) du = \left[ \frac{u^3}{3} - u \right]_1^{\sqrt{2}}$$

$$= \left[ \frac{2\sqrt{2}}{3} - \sqrt{2} \right] - \left[ \frac{1}{3} - 1 \right] = \frac{2}{3} - \frac{\sqrt{2}}{3}$$

14. Find the area between the curves  $x = y^2 - 1$  and  $y = 5 - x$ .

a. (4 pts) Graph the curves.



b. (4 pts) Set up the integral(s) for the area.

$$y^2 - 1 = 5 - y \quad y^2 + y - 6 = 0 \quad y = -3, 2$$

$$A = \int_{-3}^2 ((5 - y) - (y^2 - 1)) dy$$

c. (2 pts) Compute the area.

$$A = \int_{-3}^2 (-y^2 - y + 6) dy = \left[ -\frac{y^3}{3} - \frac{y^2}{2} + 6y \right]_{-3}^2 = \left[ -\frac{8}{3} - 2 + 12 \right] - \left[ 9 - \frac{9}{2} - 18 \right] = \frac{125}{6}$$

15. Compute  $\int x^4 \ln x dx$ .

$$u = \ln x \quad dv = x^4 dx$$

$$du = \frac{1}{x} dx \quad v = \frac{x^5}{5}$$

$$\int x^4 \ln x dx = \frac{x^5}{5} \ln x - \int \frac{x^5}{5} \frac{1}{x} dx = \frac{x^5}{5} \ln x - \frac{x^5}{25} + C$$