Student (Print) ______________________________ Section __________

Last, First Middle

Student (Sign) ______________________________

Student ID ______________________________

Instructor ______________________________

MATH 152

Exam 2

Fall 2000

Test Form A

1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Formulas:

$$M_n = \Delta x\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$$

$$T_n = \frac{\Delta x}{2}[f(x_o) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$$S_n = \frac{\Delta x}{3}[f(x_o) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$|E_M| < \frac{K(b-a)^3}{24n^2} \quad \text{where} \quad K > f''(x) \quad \text{for all } x \text{ in } [a,b]$$

$$|E_T| < \frac{K(b-a)^3}{12n^2} \quad \text{where} \quad K > f''(x) \quad \text{for all } x \text{ in } [a,b]$$

$$|E_S| < \frac{K(b-a)^5}{180n^4} \quad \text{where} \quad K > f^{(4)}(x) \quad \text{for all } x \text{ in } [a,b]$$

$$\int \ln x\, dx = x\ln x - x + C$$

$$\int \tan x\, dx = -\ln|\cos x| + C$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Calculate the x-component (or coordinate) of the center of mass of a plate with uniform density ρ and whose shape is the quarter circle of radius 3 given by $0 \le x \le 3$ and $0 \le y \le \sqrt{9-x^2}$.

a. $\frac{\pi}{4}$
b. $\frac{9\pi}{4}\rho$
c. 9ρ
d. $\frac{8}{\pi}$
e. $\frac{4}{\pi}$

2. Which of the following gives the Trapezoid Rule approximation to $\int_2^4 \frac{1}{x}\,dx$ with $n = 4$?

a. $\frac{1}{4}\left[\frac{1}{4}+\frac{2}{5}+\frac{1}{3}+\frac{2}{7}+\frac{1}{8}\right]$
b. $\frac{1}{4}\left[\frac{1}{2}+\frac{2}{5}+\frac{1}{3}+\frac{2}{7}+\frac{1}{4}\right]$
c. $\frac{1}{4}\left[\frac{1}{2}+\frac{4}{5}+\frac{2}{3}+\frac{4}{7}+\frac{1}{4}\right]$
d. $\frac{1}{2}\left[\ln 2+\ln\frac{3}{2}+\ln 3+\ln\frac{7}{2}+\ln 4\right]$
e. $\frac{1}{4}\left[\ln 2+2\ln\frac{3}{2}+2\ln 3+2\ln\frac{7}{2}+\ln 4\right]$

3. If you use the Trapezoid Rule with $n = 4$ to approximate $\int_2^4 \frac{1}{x}\,dx$, which of the following is the smallest upper bound on the error $|E_T|$ in the approximation?

a. $\frac{1}{12}$
b. $\frac{1}{24}$
c. $\frac{1}{48}$
d. $\frac{1}{96}$
e. $\frac{1}{192}$

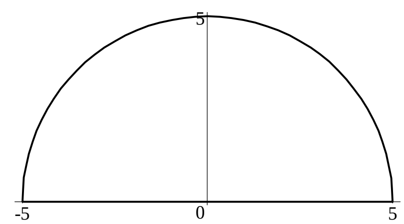
4. An integrating factor for the differential equation $\frac{dy}{dx} = x\sin 2x + y\tan x$ is $I(x) =$

a. $|\cos x|$
b. $e^{-\sec^2 x}$
c. $e^{-|\tan x|}$
d. $-|\sec x|$
e. $e^{|\cos x|}$

5. The parametric curve $x = e^t - t, \quad y = 4e^{t/2}$ for $0 \le t \le 2$ is rotated about the x-axis. Which integral gives the area of the surface of revolution?
HINT: Look for a perfect square.

a. $\int_0^2 2\pi(e^t - t)(e^{2t} + 2e^t + 1)\,dt$
b. $\int_0^2 2\pi(e^t - t)(e^t + 1)\,dt$
c. $\int_0^2 8\pi e^{t/2}(e^{2t} + 2e^t + 1)\,dt$
d. $\int_0^2 8\pi e^{t/2}(e^t + 1)\,dt$
e. $\int_0^2 8\pi e^{t/2}\sqrt{e^t + 1}\,dt$

6. A tank is completely filled with water. The end is a vertical semicircle with radius 5 m as shown. Which integral gives the hydrostatic force on this end of the tank?
The density of water is $1000\,\frac{\text{kg}}{\text{m}^3}$ and $g = 9.8\frac{\text{m}}{\text{sec}^2}$.



a. $9800\int_0^5 (5 - y)2\sqrt{25 - y^2}\,dy$
b. $9800\int_0^5 (5 + y)\sqrt{25 - y^2}\,dy$
c. $9800\int_0^5 (5 - y)(25 - y^2)\,dy$
d. $9800\int_0^5 (5 + y)2\sqrt{25 - y^2}\,dy$
e. $9800\int_0^5 (5 + y)(25 - y^2)\,dy$

7. Compute $\int_e^\infty \frac{1}{x\ln x}\,dx$.

a. 0
b. $\frac{1}{e}$
c. 1
d. e
e. ∞

8. Compute $\lim_{n\to\infty} \frac{\ln(2+e^n)}{3n}$

a. 0
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. $\frac{\ln 2}{3}$
e. ∞

9. After separating variables, we can solve the differential equation $\frac{dy}{dt} = ty + \frac{t}{y}$ by solving

a. $\int \left(\frac{1}{y} + y\right) dy = \int t\,dt$
b. $\int \frac{y}{y^2+1}\,dy = \int t\,dt$
c. $\int (y^2 + y)\,dy = \int (t+1)\,dt$
d. $\int \frac{y^2+1}{y}\,\frac{1}{dy} = \int \frac{t}{dt}$
e. None of the above, the differential equation is not separable.

10. Find the arc length of the parametric curve $x = 4\cos t,\quad y = 4\sin t$ for $\frac{\pi}{6} \le t \le \frac{\pi}{3}$.

a. $\frac{\pi}{3}$
b. $\frac{2\pi}{3}$
c. $\frac{\pi}{2}$
d. $\frac{4\pi}{3}$
e. $\frac{3\pi}{4}$

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

You may not use a calculator.

11. Determine whether each of the following sequences converges or diverges. If it converges, find the limit. Fully justify your answers.

a. $\lim_{n\to\infty} \sin\left(\frac{n\pi}{2}\right)$ Circle one: Converges Diverges

Explain:

b. $\lim_{n\to\infty} \frac{\cos^2 n}{2^n}$ Circle one: Converges Diverges

Explain:

12. Determine whether each of the following integrals converges or diverges. If it converges, find its value.
Hint: Partial fractions

a. $\int_1^\infty \frac{1}{x + 2x^2}\, dx$ Circle one: Converges Diverges

Explain:

b. $\int_0^1 \frac{1}{x + 2x^2}\, dx$ Circle one: Converges Diverges

Explain:

13. A tank contains 20 lb of salt mixed with 50 gal of water. Salt water containing 2 lb of salt per gal is added to the tank at the rate of 5 gal per min. The tank is kept thoroughly mixed and drains at the same rate.

a. Write out the differential equation and the initial condition for $S(t)$, the number of lbs of salt in the tank at time t.

b. Solve the initial value problem.

c. How much salt is in the tank after 10 hours?

14. Compute the surface area of the surface obtained by rotating the curve $x = 1 + 2y^2$ for $1 \leq y \leq 2$ about the x-axis?

15. Solve the initial value problem

$$x\frac{dy}{dx} + 2y = \frac{\cos x}{x} \qquad \text{with} \qquad y(\pi) = 1.$$