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 Last, First Middle

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Student ID _____

Instructor _____

Section _____

MATH 152
 Exam 2
 Fall 2000
 Test Form B

1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Formulas:

$$M_n = \Delta x \left[f\left(\frac{x_0 + x_1}{2}\right) + f\left(\frac{x_1 + x_2}{2}\right) + \dots + f\left(\frac{x_{n-1} + x_n}{2}\right) \right]$$

$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$|E_M| < \frac{K(b-a)^3}{24n^2} \quad \text{where } K > f''(x) \quad \text{for all } x \text{ in } [a, b]$$

$$|E_T| < \frac{K(b-a)^3}{12n^2} \quad \text{where } K > f''(x) \quad \text{for all } x \text{ in } [a, b]$$

$$|E_S| < \frac{K(b-a)^5}{180n^4} \quad \text{where } K > f^{(4)}(x) \quad \text{for all } x \text{ in } [a, b]$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int \tan x dx = -\ln|\cos x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Compute $\int_e^\infty \frac{1}{x \ln x} dx$.

- a. 0
- b. $\frac{1}{e}$
- c. 1
- d. e
- e. ∞

2. After separating variables, we can solve the differential equation $\frac{dy}{dt} = ty + \frac{t}{y}$ by solving

- a. $\int \left(\frac{1}{y} + y \right) dy = \int t dt$
- b. $\int \frac{y}{y^2 + 1} dy = \int t dt$
- c. $\int (y^2 + y) dy = \int (t + 1) dt$
- d. $\int \frac{y^2 + 1}{y} \frac{1}{dy} = \int \frac{t}{dt}$
- e. None of the above, the differential equation is not separable.

3. Compute $\lim_{n \rightarrow \infty} \frac{\ln(2 + e^n)}{3n}$

- a. 0
- b. $\frac{1}{3}$
- c. $\frac{2}{3}$
- d. $\frac{\ln 2}{3}$
- e. ∞

4. Which of the following gives the Trapezoid Rule approximation to $\int_2^4 \frac{1}{x} dx$ with $n = 4$?

- a. $\frac{1}{4} \left[\frac{1}{4} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7} + \frac{1}{8} \right]$
- b. $\frac{1}{4} \left[\frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7} + \frac{1}{4} \right]$
- c. $\frac{1}{4} \left[\frac{1}{2} + \frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{4} \right]$
- d. $\frac{1}{2} \left[\ln 2 + \ln \frac{3}{2} + \ln 3 + \ln \frac{7}{2} + \ln 4 \right]$
- e. $\frac{1}{4} \left[\ln 2 + 2 \ln \frac{3}{2} + 2 \ln 3 + 2 \ln \frac{7}{2} + \ln 4 \right]$

5. If you use the Trapezoid Rule with $n = 4$ to approximate $\int_2^4 \frac{1}{x} dx$, which of the following is the smallest upper bound on the error $|E_T|$ in the approximation?

- a. $\frac{1}{12}$
- b. $\frac{1}{24}$
- c. $\frac{1}{48}$
- d. $\frac{1}{96}$
- e. $\frac{1}{192}$

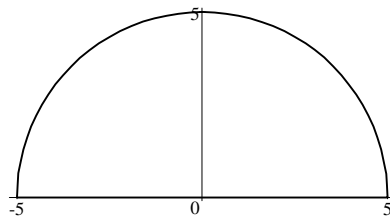
6. An integrating factor for the differential equation $\frac{dy}{dx} = x \sin 2x + y \tan x$ is $I(x) =$

- a. $|\cos x|$
- b. $e^{-\sec^2 x}$
- c. $e^{-|\tan x|}$
- d. $-|\sec x|$
- e. $e^{|\cos x|}$

7. Find the arc length of the parametric curve $x = 4 \cos t$, $y = 4 \sin t$ for $\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$.

- a. $\frac{\pi}{3}$
- b. $\frac{2\pi}{3}$
- c. $\frac{\pi}{2}$
- d. $\frac{4\pi}{3}$
- e. $\frac{3\pi}{4}$

8. A tank is completely filled with water. The end is a vertical semicircle with radius 5 m as shown. Which integral gives the hydrostatic force on this end of the tank?



The density of water is $1000 \frac{\text{kg}}{\text{m}^3}$ and $g = 9.8 \frac{\text{m}}{\text{sec}^2}$.

- a. $9800 \int_0^5 (5 - y)2\sqrt{25 - y^2} dy$
 b. $9800 \int_0^5 (5 + y) \sqrt{25 - y^2} dy$
 c. $9800 \int_0^5 (5 - y)(25 - y^2) dy$
 d. $9800 \int_0^5 (5 + y)2\sqrt{25 - y^2} dy$
 e. $9800 \int_0^5 (5 + y)(25 - y^2) dy$
9. The parametric curve $x = e^t - t$, $y = 4e^{t/2}$ for $0 \leq t \leq 2$ is rotated about the x -axis. Which integral gives the area of the surface of revolution?
 HINT: Look for a perfect square.
- a. $\int_0^2 2\pi(e^t - t)(e^{2t} + 2e^t + 1) dt$
 b. $\int_0^2 2\pi(e^t - t)(e^t + 1) dt$
 c. $\int_0^2 8\pi e^{t/2}(e^{2t} + 2e^t + 1) dt$
 d. $\int_0^2 8\pi e^{t/2}(e^t + 1) dt$
 e. $\int_0^2 8\pi e^{t/2} \sqrt{e^t + 1} dt$
10. Calculate the x -component (or coordinate) of the center of mass of a plate with uniform density ρ and whose shape is the quarter circle of radius 3 given by $0 \leq x \leq 3$ and $0 \leq y \leq \sqrt{9 - x^2}$.
- a. $\frac{\pi}{4}$
 b. $\frac{9\pi}{4}\rho$
 c. 9ρ
 d. $\frac{8}{\pi}$
 e. $\frac{4}{\pi}$

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

You may not use a calculator.

11. Determine whether each of the following integrals converges or diverges. If it converges, find its value.

Hint: Partial fractions

a. $\int_1^{\infty} \frac{2}{2x+x^2} dx$
Explain:

Circle one: Converges Diverges

b. $\int_0^1 \frac{2}{2x+x^2} dx$
Explain:

Circle one: Converges Diverges

12. Determine whether each of the following sequences converges or diverges. If it converges, find the limit. Fully justify your answers.

a. $\lim_{n \rightarrow \infty} \cos\left(\frac{n\pi}{2}\right)$
Explain:

Circle one: Converges Diverges

b. $\lim_{n \rightarrow \infty} \frac{\sin^2 n}{2^n}$
Explain:

Circle one: Converges Diverges

13. Solve the initial value problem

$$x \frac{dy}{dx} + 2y = \frac{\cos x}{x} \quad \text{with} \quad y(\pi) = 2.$$

14. Compute the surface area of the surface obtained by rotating the curve $x = 1 + y^2$ for $1 \leq y \leq 2$ about the x -axis?

15. A tank contains 40 lb of salt mixed with 60 gal of water. Salt water containing 4 lb of salt per gal is added to the tank at the rate of 3 gal per min. The tank is kept thoroughly mixed and drains at the same rate.
- Write out the differential equation and the initial condition for $S(t)$, the number of lbs of salt in the tank at time t .
 - Solve the initial value problem.
 - How much salt is in the tank after 20 hours?