Student (Print) _____ Section _____

Last, First Middle

Student (Sign) _____

Student ID _____

Instructor _____

MATH 152

Exam 3

Fall 2000

Test Form A

1-9	/45
10	/10
11	/15
12	/10
13	/10
14	/10
TOTAL	

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Compute $\sum_{n=0}^{\infty} 3^n 5^{2-n}$

a. $\frac{25}{3}$

b. $\frac{125}{3}$

c. $\frac{125}{2}$

d. $\frac{75}{2}$

e. ∞

2. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is

a. absolutely convergent

b. convergent but not absolutely convergent

c. divergent

d. none of the above

3. Determine the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{n+1}$.

a. $(2,4]$

b. $[2,4)$

c. $(-4,-2]$

d. $[-4,-2)$

e. $(-\infty,\infty)$

4. Find the Maclaurin series for $f(x) = \sin 2x$.

a. $2x - \frac{2^3x^3}{3!} + \frac{2^5x^5}{5!} - \frac{2^7x^7}{7!} + \cdots$

b. $2x - \frac{2x^3}{3!} + \frac{2x^5}{5!} - \frac{2x^7}{7!} + \cdots$

c. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

d. $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$

e. $2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \frac{2x^7}{7!} + \cdots$

5. Find the 3^{rd} degree Taylor polynomial for $f(x) = e^{-2x}$ about $x = 1$.

a. $e^{-2} + 2e^{-2}(x-1) + 2e^{-2}(x-1)^2 + \frac{4}{3}e^{-2}(x-1)^3$

b. $e^{-2} - 2e^{-2}(x-1) + 2e^{-2}(x-1)^2 - \frac{4}{3}e^{-2}(x-1)^3$

c. $e^{-2} + 2e^{-2}(x-1) + 4e^{-2}(x-1)^2 + 8e^{-2}(x-1)^3$

d. $e^{-2} - 2e^{-2}(x-1) + 4e^{-2}(x-1)^2 - 8e^{-2}(x-1)^3$

e. $e^{-2} - 2e^{-2}(x-1) - 4e^{-2}(x-1)^2 - 8e^{-2}(x-1)^3$

6. Compute $\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^{n-1}$. HINT: Differentiate $\sum_{n=0}^{\infty} x^n$.

a. -4

b. -2

c. $\frac{4}{9}$

d. 2

e. 4

7. Find a unit vector which is orthogonal to the plane containing the points $P = (1,0,0)$, $Q = (0,2,0)$, and $R = (0,0,3)$.

a. $\left(\frac{6}{11}, \frac{-3}{11}, \frac{2}{11}\right)$

b. $(6,-3,2)$

c. $(6,3,2)$

d. $\left(\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}\right)$

e. $\left(\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$

8. Find the equation of the circle for which one diameter has endpoints $P = (1,1)$ and $Q = (7,9)$.

a. $(x+3)^2 + (y+4)^2 = 25$

b. $(x+3)^2 + (y+4)^2 = 5$

c. $(x-4)^2 + (y-5)^2 = 25$

d. $(x-3)^2 + (y-4)^2 = 5$

e. $(x+4)^2 + (y+5)^2 = 25$

9. Consider the triangle with vertices $A = (0,0,0)$, $B = (1,1,0)$ and $C = (0,2,2)$. Find the angle at A.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

Part II: Work Out

Show all your work. Partial credit will be given.

You may not use a calculator.

10. (10 points)

a. Find the power series representation for $f(x) = \frac{1}{1+x^2}$ centered at $x = 0$ and its radius of convergence.

b. Find the power series representation for $f(x) = x\tan^{-1}x$ centered at $x = 0$ and its radius of convergence. HINT: $\tan^{-1}x = \int_0^x \frac{1}{1+t^2}dt$

11. (15 points) Determine if each of the following series converges or diverges. Say why. Be sure to name or quote the test(s) you use and check out all requirements of the test.

a. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ Circle one: Converges Diverges

Explain:

b. $\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$ Circle one: Converges Diverges

Explain:

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^{n-1}}{n^2 4^n}$ Circle one: Converges Diverges

Explain:

12. (10 points) Determine if each of the following series converges or diverges. Say why. Be sure to name or quote the test(s) you use and check out all requirements of the test. **If it converges**, find the sum. **If it diverges**, does it diverge to $+\infty$, $-\infty$ or neither?

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+\sqrt[3]{n}}$ Circle one: Converges Diverges

Explain:

b. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-1}{n} - \frac{n}{n+1} \right)$ Circle one: Converges Diverges

Explain:

13. (10 points) Find the volume of the parallelepiped with adjacent edges $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$, where $P = (1,1,1)$, $Q = (2,3,1)$, $R = (4,1,5)$ and $S = (1,3,4)$.

14. (10 points) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^{1/3}\,3^n}$. Be sure to show how you checked the convergence at the endpoints.