

Student (Print) _____
Last, First Middle
Student (Sign) _____
Student ID _____
Instructor _____

Section _____

MATH 152
Exam 3
Fall 2000
Test Form B

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

1-9	/45
10	/10
11	/15
12	/10
13	/10
14	/10
TOTAL	

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is

- a. absolutely convergent
- b. convergent but not absolutely convergent
- c. divergent
- d. none of the above

2. Determine the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{n+1}$.

- a. $(2, 4]$
- b. $[2, 4)$
- c. $(-4, -2]$
- d. $[-4, -2)$
- e. $(-\infty, \infty)$

3. Compute $\sum_{n=0}^{\infty} 3^n 5^{2-n}$

- a. $\frac{25}{3}$
- b. $\frac{125}{3}$
- c. $\frac{125}{2}$
- d. $\frac{75}{2}$
- e. ∞

4. Find the 3rd degree Taylor polynomial for $f(x) = e^{-2x}$ about $x = 1$.

- a. $e^{-2} + 2e^{-2}(x-1) + 2e^{-2}(x-1)^2 + \frac{4}{3}e^{-2}(x-1)^3$
- b. $e^{-2} - 2e^{-2}(x-1) + 2e^{-2}(x-1)^2 - \frac{4}{3}e^{-2}(x-1)^3$
- c. $e^{-2} + 2e^{-2}(x-1) + 4e^{-2}(x-1)^2 + 8e^{-2}(x-1)^3$
- d. $e^{-2} - 2e^{-2}(x-1) + 4e^{-2}(x-1)^2 - 8e^{-2}(x-1)^3$
- e. $e^{-2} - 2e^{-2}(x-1) - 4e^{-2}(x-1)^2 - 8e^{-2}(x-1)^3$

5. Compute $\sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^{n-1}$. HINT: Differentiate $\sum_{n=0}^{\infty} x^n$.

- a. -4
- b. -2
- c. $\frac{4}{9}$
- d. 2
- e. 4

6. Find the Maclaurin series for $f(x) = \sin 2x$.

- a. $2x - \frac{2^3x^3}{3!} + \frac{2^5x^5}{5!} - \frac{2^7x^7}{7!} + \dots$
- b. $2x - \frac{2x^3}{3!} + \frac{2x^5}{5!} - \frac{2x^7}{7!} + \dots$
- c. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
- d. $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$
- e. $2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \frac{2x^7}{7!} + \dots$

7. Find the equation of the circle for which one diameter has endpoints $P = (1,1)$ and $Q = (7,9)$.
- $(x+3)^2 + (y+4)^2 = 25$
 - $(x+3)^2 + (y+4)^2 = 5$
 - $(x-4)^2 + (y-5)^2 = 25$
 - $(x-3)^2 + (y-4)^2 = 5$
 - $(x+4)^2 + (y+5)^2 = 25$
8. Consider the triangle with vertices $A = (0,0,0)$, $B = (1,1,0)$ and $C = (0,2,2)$. Find the angle at A.
- 0°
 - 30°
 - 45°
 - 60°
 - 90°
9. Find a unit vector which is orthogonal to the plane containing the points $P = (1,0,0)$, $Q = (0,2,0)$, and $R = (0,0,3)$.
- $\left(\frac{6}{11}, \frac{-3}{11}, \frac{2}{11}\right)$
 - $(6, -3, 2)$
 - $(6, 3, 2)$
 - $\left(\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}\right)$
 - $\left(\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$

Part II: Work Out

Show all your work. Partial credit will be given.

You may not use a calculator.

10. (10 points) Determine if each of the following series converges or diverges. Say why. Be sure to name or quote the test(s) you use and check out all requirements of the test. **If it converges**, find the sum. **If it diverges**, does it diverge to $+\infty$, $-\infty$ or neither?

a.
$$\sum_{n=1}^{\infty} \left(\frac{n-1}{n} - \frac{n}{n+1} \right)$$

Circle one: Converges Diverges

Explain:

b.
$$\sum_{n=1}^{\infty} \frac{1}{1 + \sqrt[3]{n}}$$

Circle one: Converges Diverges

Explain:

11. (15 points) Determine if each of the following series converges or diverges. Say why. Be sure to name or quote the test(s) you use and check out all requirements of the test.

a. $\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

Circle one: Converges Diverges

Explain:

b. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^{n-1}}{n^2 4^n}$

Circle one: Converges Diverges

Explain:

c. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

Circle one: Converges Diverges

Explain:

12. (10 points)

a. Find the power series representation for $f(x) = \frac{1}{1+x^2}$ centered at $x = 0$ and its radius of convergence.

b. Find the power series representation for $f(x) = x \tan^{-1} x$ centered at $x = 0$ and its radius of convergence. HINT: $\tan^{-1} x = \int_0^x \frac{1}{1+t^2} dt$

13. (10 points) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^{1/3} 3^n}$. Be sure to show how you checked the convergence at the endpoints.

14. (10 points) Find the volume of the parallelepiped with adjacent edges \vec{PQ} , \vec{PR} and \vec{PS} , where $P = (1, 1, 1)$, $Q = (2, 3, 1)$, $R = (4, 1, 5)$ and $S = (1, 3, 4)$.