

Name _____ ID _____ Sec _____

(Print) Last, First Middle

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MATH 152
Sections 510,511

Final Exam

Fall 2000
P. Yasskin

Multiple Choice: (4 points each)

1-12	/48
13	/13
14	/13
15	/13
16	/13

1. Compute $\lim_{n \rightarrow \infty} \frac{2^{n+1}}{n!}$

- a. 0
- b. 1
- c. 2
- d. 4
- e. ∞

2. Compute $\int_0^{\pi/4} \tan \theta \sec^2 \theta d\theta$

- a. $-\frac{1}{2}$
- b. $-\frac{1}{3}$
- c. 0
- d. $\frac{1}{3}$
- e. $\frac{1}{2}$

3. The region below $y = \frac{1}{x}$ above the x -axis between $x = 1$ and $x = 2$ is rotated about the y -axis. Find the volume of the solid of revolution.

- a. 4π
- b. 2π
- c. π
- d. $\frac{\pi}{2}$
- e. $\frac{\pi}{4}$

4. Compute $\int_0^{1/2} x \cos(\pi x) dx$

- a. $\frac{1}{2\pi} - \frac{1}{\pi^2}$
- b. $\frac{1}{\pi^2} + \frac{1}{2\pi}$
- c. $\pi^2 - 2\pi$
- d. $-2\pi - \pi^2$
- e. $\frac{1}{\pi^2} - \frac{1}{2\pi}$

5. Find the volume of the parallelepiped whose adjacent edges are

$$\vec{u} = (2, -1, 3), \quad \vec{v} = (0, 1, 2) \quad \text{and} \quad \vec{w} = (3, 0, 1).$$

- a. 1
- b. -1
- c. 13
- d. -13
- e. $10\sqrt{7}$

6. Compute $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1 - x^2}{x^4}$

- a. $-\frac{1}{6}$
- b. $-\frac{1}{2}$
- c. 0
- d. $\frac{1}{2}$
- e. $\frac{1}{6}$

7. A 50 gal tank is initially filled with salt water whose concentration is $0.6 \frac{\text{lb sugar}}{\text{gal water}}$. Pure water is added at the rate of $5 \frac{\text{gal}}{\text{hr}}$. The mixture is kept well mixed and drained at the rate of $5 \frac{\text{gal}}{\text{hr}}$. How many hours does it take until the concentration drops to $0.3 \frac{\text{lb sugar}}{\text{gal water}}$.

- a. $T = -10 \ln \frac{1}{2}$
- b. $T = 10 \ln \frac{1}{2}$
- c. $T = \ln 5$
- d. $T = -\frac{1}{10} \ln 2$
- e. $T = \frac{1}{10} \ln 2$

8. Compute $\int_2^3 \frac{1}{(x-2)^{4/3}} dx$

- a. $-\infty$
- b. -3
- c. -1
- d. 3
- e. ∞

9. The curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated about the x -axis. Which formula will give the surface area of the surface of revolution?

- a. $A = \int_0^\pi 2\pi x \sqrt{1 + \cos^2 x} dx$
- b. $A = \int_0^\pi \sqrt{1 + \cos^2 x} dx$
- c. $A = \int_0^\pi 2\pi \sin x \sqrt{1 + \cos^2 x} dx$
- d. $A = \int_0^\pi 2\pi x \sqrt{1 + \sin^2 x} dx$
- e. $A = \int_0^\pi \sqrt{1 + \sin^2 x} dx$

10. Find the solution of the differential equation $x \frac{dy}{dx} - 2y = 4x^3$ satisfying the initial condition $y(2) = 4$.

a. $y = 4x^3 - \frac{127}{8}x^2$

b. $y = 4x^3 - 28$

c. $y = 4x^3 - 7x^2$

d. $y = \frac{4}{5}x^3 - \frac{12}{5}$

e. $y = \frac{4}{5}x^3 - \frac{48}{5x^2}$

11. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)^2}{3^n} (x-2)^n$.

a. 0

b. $\frac{1}{2}$

c. 2

d. $\frac{1}{3}$

e. 3

12. Find an equation of the plane perpendicular to the line

$$x = 2 - 2t \quad y = -1 + 3t \quad z = 3 + t$$

which contains the point $P = (1, 2, 3)$

a. $2x - y + 3z = 9$

b. $-2x + 3y + z = 7$

c. $x + 2y + 3z = 14$

d. $2x + y + z = 7$

e. $2x + 2y + z = 9$

Work Out (13 points each)

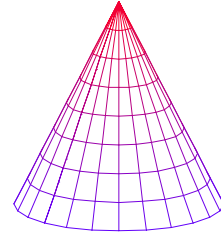
Show all your work. Partial credit will be given. You may not use a calculator.

13. Compute $\int \frac{32}{(x-2)(x+2)(x^2+4)} dx$

14. A water tank has the shape of a cone with the vertex at the top. The cone is 8 m tall and has a base which is a circle of radius 4 m. The cone is filled with water to a depth of 5 m. How much work is needed to pump the water out the top of the tank?

(The density of water is $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ and the acceleration of gravity is $g = 9.8 \frac{\text{m}}{\text{sec}^2}$,

but you may leave your answer as a multiple of ρg .)



15. Determine if each of the following series converges or diverges. Say why. Be sure to name or quote the test(s) you use and check out all requirements of the test. **If it converges**, find the sum. **If it diverges**, does it diverge to $+\infty$, $-\infty$ or neither?

a.
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

Explain:

Circle one: Converges Diverges

b.
$$\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{n!}$$

Explain:

Circle one: Converges Diverges

16. Find the arclength of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ for $1 \leq y \leq 3$.