PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 5 points: NO partial credit will be given. The use of a calculator is prohibited.

1. Find the area of the region bounded between the curves $y = 0$ and $y = \sin x$ from $x = \pi/4$ to $x = \pi/2$.

   (a) 1

   (b) $\frac{\sqrt{2}}{2}$

   (c) $\sqrt{2}$

   (d) $\frac{\sqrt{3}}{3}$

   (e) $\sqrt{3}$

2. Find the average value of the function $g(x) = \sqrt{1 + 2x}$ on the interval $[1, 4]$.

   (a) $9 - \sqrt{3}$

   (b) $18 - 2\sqrt{3}$

   (c) $6 - \frac{2\sqrt{3}}{3}$

   (d) $3 - \frac{\sqrt{3}}{3}$

   (e) $\frac{3}{2} - \frac{\sqrt{3}}{6}$

3. The ellipse $\frac{x^2}{4} + \frac{y^2}{36} = 1$ is revolved about the x-axis. Which integral gives the volume of the resulting ellipsoid?

   (a) $\pi \int_{-2}^{2} (36 - 9x^2)\, dx$

   (b) $\pi \int_{-6}^{6} (36 - 9x^2)\, dx$

   (c) $2\pi \int_{-2}^{2} x\sqrt{36 - 9x^2}\, dx$

   (d) $2\pi \int_{-6}^{6} x\sqrt{36 - 9x^2}\, dx$

   (e) $\pi \int_{-2}^{2} (36 - 9x^2)^2\, dx$

4. Using a trigonometric substitution, the integral $\displaystyle\int \frac{x^2}{\sqrt{x^2+25}}\,dx$ becomes:

(a) $25\displaystyle\int (\tan^2\theta)(\sec\theta)\,d\theta$

(b) $5\displaystyle\int (\tan^2\theta)(\sec\theta)\,d\theta$

(c) $25\displaystyle\int \frac{\tan^2\theta}{\sec\theta}\,d\theta$

(d) $5\displaystyle\int \frac{\tan^2\theta}{\sec\theta}\,d\theta$

(e) $25\displaystyle\int \sin^2\theta\,d\theta$

5. Compute $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx$

(a) 2

(b) 4

(c) 0

(d) 2π

(e) 4π

6. If $F(0)=1$ and $F(3)=5$, then $\displaystyle\int_0^3 F'(x)\,dx =$

(a) 8

(b) 6

(c) 5

(d) 4

(e) Can't be determined from the given information.

7. Which of these expressions represents the area between the curves $y = x^2$ and $y = 6 - x$ from $x = 0$ to $x = 3$?

(a) $\int_0^3 (6 - x - x^2)\, dx$

(b) $\int_0^3 (x^2 + x - 6)\, dx$

(c) $\int_0^2 (6 - x - x^2)\, dx + \int_2^3 (x^2 + x - 6)\, dx$

(d) $\int_0^2 (x^2 + x - 6)\, dx + \int_2^3 (6 - x - x^2)\, dx$

(e) $\int_0^1 (6 - x - x^2)\, dx + \int_1^3 (x^2 + x - 6)\, dx$

8. The base of a solid is the ellipse $x^2 + \dfrac{y^2}{9} = 1$. Cross-sections perpendicular to the y-axis are squares. Find the volume.

(a) $\int_0^3 4\left(1 - \dfrac{y^2}{9}\right) dy$

(b) $\int_{-3}^3 4\left(1 - \dfrac{y^2}{9}\right) dy$

(c) $\int_{-3}^3 \left(1 - \dfrac{y^2}{9}\right) dy$

(d) $\int_{-1}^1 4\left(1 - \dfrac{y^2}{9}\right) dy$

(e) $\int_0^1 \left(1 - \dfrac{y^2}{9}\right) dy$

9. Evaluate $\int_0^1 xe^{-x}\,dx$

(a) 1

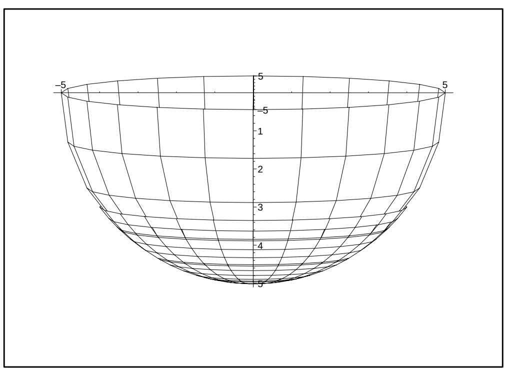
(b) $1+\frac{1}{e}$

(c) $1-\frac{1}{e}$

(d) $1+\frac{2}{e}$

(e) $1-\frac{2}{e}$

10. A water tank has the shape of a hemisphere with radius 5 meters. It is filled with water to a height of 2 meters. Find the work in Joules required to empty the tank by pumping all of the water to the top of the tank. Here, ρ is the density of water in kilograms/(meter)^3 and g is the acceleration of gravity in meters/(second)^2.



(a) $\frac{625\pi\rho g}{4}$

(b) $\frac{52\pi\rho g}{3}$

(c) $64\pi\rho g$

(d) $36\pi\rho g$

(e) $72\pi\rho g$

PART 2: WORK-OUT PROBLEMS

Each problem is worth 10 points; partial credit is possible. The use of a calculator is prohibited. **SHOW ALL WORK!**

11. Evaluate $\int (\ln x)^2 \, dx$

12. Compute $\int_2^{2\sqrt{2}} \frac{\sqrt{x^2 - 4}}{x} \, dx$

13. Consider the region R bounded by the curves $y = 4 - x^2$ and $y = -3x$.

(a) Set up and evaluate an integral with respect to x that gives the **area** of the region R. (6 pts)

(b) Find, but **DO NOT** evaluate an expression that involves integration with respect to y that represents the **area** of the region R. (4 pts)

14. Consider the region R bounded between the curves $y = x^3$ and $y = 2x^2$.

(a) Find the **volume** of the solid obtained by revolving the region R about the x-axis. (5 pts)

(b) Find the **volume** of the solid obtained by revolving the region R about the y-axis. (5 pts)

15. Evaluate $\int (\sec^5 x)(\tan^3 x)\, dx$