VERSION B SOLUTIONS

PART 1: MULTIPLE-CHOICE PROBLEMS

1. The center of mass is located at

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2)(-2) + (4)(1) + (6)(4)}{2 + 4 + 6} = \frac{24}{12} = 2 \ .$$

The correct answer is **(c)**.

2. The area of the surface generated by rotating the curve $y = x^2$ from $x = 0$ to $x = \sqrt{2}$ about the y-axis is

$$\int_0^{\sqrt{2}} 2\pi x\sqrt{1 + (dy/dx)^2}\, dx = \int_0^{\sqrt{2}} 2\pi x\sqrt{1 + (2x)^2}\, dx = \int_0^{\sqrt{2}} 2\pi x\sqrt{1 + 4x^2}\, dx \ .$$

The correct answer is **(e)**.

3. Now

$$\frac{2}{x(x+2)} = \frac{1}{x} - \frac{1}{x+2}$$

$$\int \frac{2}{x(x+2)}\, dx = \int \frac{1}{x}\, dx - \int \frac{1}{x+2}\, dx = \ln|x| - \ln|x+2| + C \ .$$

The correct answer is **(c)**.

4. The partition of $[1, 13]$ when $n = 3$ is $\{1, 5, 9, 13\}$. So $\Delta x = 4$ and the midpoints of the subintervals are $3, 7$ and 11. Thus, $M_3 = (4)[(1/3) + (1/7) + (1/11)]$.

The correct answer is **(b)**.

5. The length of the curve $x = 2t + t^2$, $y = 2t - t^2$ for $0 \leq t \leq 3$ is

$$\int_0^3 \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \int_0^3 \sqrt{(2 + 2t)^2 + (2 - 2t)^2}\, dt = \int_0^3 \sqrt{8(1 + t^2)}\, dt \ .$$

The correct answer is **(a)**.

6. Now

$$0 \leq \frac{\cos^2 x}{x^2 + \sqrt{x}} \leq \frac{1}{x^2}$$

for all $x \geq 1$ and $\displaystyle\int_1^\infty \frac{1}{x^2}\, dx$ converges. The correct answer is **(b)**.

7. The length of the curve $y = (2/3)x^{3/2}$ from $x = 0$ to $x = 8$ is

$$\int_0^8 \sqrt{1 + (dy/dx)^2}\, dx = \int_0^8 \sqrt{1 + (\sqrt{x})^2}\, dx = \int_0^8 \sqrt{1 + x}\, dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^8 = \frac{52}{3} .$$

The correct answer is **(d)**.

8. The differential equation is separable.

$$\begin{aligned} \frac{dy}{dx} &= xy \\ \frac{1}{y} dy &= x\, dx \\ \int \frac{1}{y}\, dy &= \int x\, dx + C \\ \ln y &= \frac{x^2}{2} + C \\ y &= e^{(x^2/2)+C} . \end{aligned}$$

Now $y(0) = 2$, so $C = \ln 2$ and we have that $y(x) = 2e^{(x^2/2)}$. Thus, $y(2) = 2e^2$ and the correct answer is **(a)**.

9. If $y(t)$ is the amount of salt (in kilograms) in the tank after t minutes, then

$$\frac{dy}{dt} = \text{rate in} - \text{rate out} = 0 - \left(\frac{y(t)}{1000} \frac{\text{kg}}{\text{L}}\right)\left(20 \frac{\text{L}}{\text{min}}\right) = -\frac{y(t)}{50} \frac{\text{kg}}{\text{min}} .$$

Now $y(0) = 10$ kg and the correct answer is **(d)**.

10. The hydrostatic force (in Newtons) on one side of the vertical rectangular plate is

$$\rho g \int_9^{12} 10y\, dy = 9800 \int_9^{12} 10y\, dy = 9800 \times 315 .$$

The correct answer is **(e)**.

PART 2: WORK-OUT PROBLEMS

11. The differential equation $\dfrac{dy}{dx} - (2/x)y = x$ has integrating factor $I(x) = e^{\int -(2/x)\,dx} = e^{-2\ln x} = x^{-2}$. Thus,

$$\begin{aligned} x^{-2}\left(\frac{dy}{dx} - (2/x)y\right) &= x^{-1} \\ \frac{d}{dx}(x^{-2}y) &= x^{-1} \\ x^{-2}y &= \ln x + C \\ y &= x^2(\ln x + C)\ . \end{aligned}$$

Now $2 = y(1) = (1)^2(\ln 1 + C) = C$, so $y = x^2(2 + \ln x)$.

12. Now

$$\begin{aligned} \frac{x+18}{x^3+9x} = \frac{x+18}{x(x^2+9)} &= \frac{A}{x} + \frac{Bx+C}{x^2+9} \\ &= \frac{Ax^2+9A+Bx^2+Cx}{x^3+9x} \\ &= \frac{(A+B)x^2+Cx+9A}{x^3+9x}\ , \end{aligned}$$

so

$$\begin{aligned} A + B &= 0 \\ C &= 1 \\ 9A &= 18 \end{aligned}$$

the solution of which is $A = 2$, $B = -2$ and $C = 1$. Thus

$$\begin{aligned} \int \frac{x+18}{x^3+9x}\,dx &= \int \left(\frac{2}{x} + \frac{1-2x}{x^2+9}\right)\,dx \\ &= \int \frac{2}{x}\,dx + \int \frac{1}{x^2+9}\,dx - \int \frac{2x}{x^2+9}\,dx \\ &= 2\ln|x| + \frac{1}{3}\tan^{-1}(x/3) - \ln(x^2+9) + C\ . \end{aligned}$$

13. The x-coordinate of the centroid of the region in the first quadrant that is bounded by the curves $y = 4 - x^2$, $y = 0$ and $x = 0$ is

$$\bar{x} = \frac{\int_0^2 x(4-x^2)\,dx}{\int_0^2 4-x^2\,dx} = \frac{\int_0^2 4x - x^3\,dx}{\int_0^2 4-x^2\,dx} = \frac{\left(2x^2 - \frac{x^4}{4}\right)\Big|_0^2}{\left(4x - \frac{x^3}{3}\right)\Big|_0^2} = \frac{3}{4} .$$

14. An integration by parts gives

$$\int_1^b xe^{-x}\,dx = -xe^{-x}\Big|_1^b + \int_1^b e^{-x}\,dx = 2e^{-1} - be^{-b} - e^{-b} .$$

Now $\lim_{b\to\infty} be^{-b} = 0$ and $\lim_{b\to\infty} e^{-b} = 0$, so

$$\int_1^\infty xe^{-x}\,dx = \lim_{b\to\infty} \int_1^b xe^{-x}\,dx = \lim_{b\to\infty} (2e^{-1} - be^{-b} - e^{-b}) = 2e^{-1} .$$

15. The partition of $[1,3]$ when $n = 6$ is $\{1, 4/3, 5/3, 2, 7/3, 8/3, 3\}$. So $\Delta x = 1/3$ and

$$\begin{aligned}\int_1^3 f(x)\,dx \approx S_6 &= \frac{\Delta x}{3}[f(1) + 4f(4/3) + 2f(5/3) + 4f(2) + 2f(7/3) + 4f(8/3) + f(3)] \\ &= \frac{1}{9}[(1/2) + 4(1/8) + 2(1/2) + 4(1/6) + 2(1/6) + 4(1/4) + (1/2)] \\ &= \frac{1}{2} .\end{aligned}$$

Now $|f^{(4)}(x)| \le 80$ for all $1 \le x \le 3$, so

$$|E_S| \le \frac{80(3-1)^5}{180(6)^4} = \frac{8}{729} .$$