PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 5 points: NO partial credit will be given. The use of calculators is prohibited.

1. As n approaches infinity, the sequence $\left\{ \dfrac{\sin 2n}{n} \right\}_{n=1}^{\infty}$

   (a) converges to 2

   (b) converges to 1

   (c) diverges to $\dfrac{1}{2}$

   (d) converges to 0

   (e) diverges by comparison to the sequence $\{1/n\}_{n=1}^{\infty}$

2. Which of the following is a *unit* vector that is orthogonal (perpendicular) to the vector $\mathbf{a} = \langle 1, -1, 2 \rangle$?

   (a) $\langle 1, -1, -1 \rangle$

   (b) $\langle 1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3} \rangle$

   (c) $\langle -1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3} \rangle$

   (d) $\langle 1/\sqrt{2}, -1/\sqrt{2}, -1/\sqrt{2} \rangle$

   (e) $\langle -1/\sqrt{2}, -1/\sqrt{2}, 1/\sqrt{3} \rangle$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}}{5^n} =$

   (a) Diverges

   (b) $\dfrac{5}{7}$

   (c) $\dfrac{14}{29}$

   (d) $\dfrac{5}{6}$

   (e) $\dfrac{1}{2}$

4. The third-degree Taylor polynomial for $f(x) = \ln x$ at $a = 1$ is

(a) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

(b) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{2}{3}(x-1)^3$

(c) $(x-1) - (x-1)^2 + 2(x-1)^3$

(d) $\ln x + \frac{x-1}{x} - \frac{(x-1)^2}{2x^2} + \frac{(x-1)^3}{2x^3}$

(e) $\ln x + \frac{x-1}{x} - \frac{(x-1)^2}{2x^2} + \frac{(x-1)^3}{3x^3}$

5. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{2n^2+2}{3n^2+1}$, then $\sum_{n=1}^{\infty} a_n =$

(a) 0

(b) $\frac{2}{3}$

(c) Diverges

(d) $\frac{3}{2}$

(e) 1

6. $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} =$

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) 0

(d) $\frac{1}{2}$

(e) Diverges

7. The infinite series $\sum_{n=1}^{\infty} \frac{n}{n^3+5}$

(a) Converges by the nth term divergence test.

(b) Diverges by the comparison test.

(c) Converges by the comparison test.

(d) Diverges by the ratio test.

(e) Converges by the ratio test.

8. The center of the sphere $x^2 + y^2 + z^2 + 6x - 8y = 0$ is located at

(a) $(0, 0, 0)$

(b) $(3, -4, 0)$

(c) $(3, 4, 0)$

(d) $(-3, 4, 0)$

(e) $(3, 0, 4)$

9. Which of the following series converges?

(a) $\sum\limits_{n = 2}^{\infty} (-1)^n \sqrt{n}$

(b) $\sum\limits_{n = 2}^{\infty} \dfrac{n^3}{n^4 + 5}$

(c) $\sum\limits_{n = 1}^{\infty} \dfrac{n!}{(n^{2002})}$

(d) $\sum\limits_{n = 2}^{\infty} \dfrac{1}{n(\ln n)^2}$

(e) $\sum\limits_{n = 1}^{\infty} \left(1 - \frac{1}{n}\right)^{-n}$

10. Which of the following series is absolutely convergent?

(a) $\sum\limits_{n=2}^{\infty}(-1)^n \dfrac{1}{n \ln n}$

(b) $\sum\limits_{n=1}^{\infty}(-1)^n \dfrac{1}{\sqrt{n}}$

(c) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^{0.998}}$

(d) $\sum\limits_{n=1}^{\infty} \dfrac{1}{n}$

(e) $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^{1.001}}$

PART 2: WORK-OUT PROBLEMS

The use of calculators is prohibited. SHOW ALL WORK!

11. Find the radius of convergence and the exact interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n(2^n)}$.

    Clearly justify your answer!

    (10 points)

12. Apply a test to determine if each of the following series converges or diverges. You *must* name the test, clearly explain why the test applies and clearly justify your conclusion.

(a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$ Circle one : Converges Diverges

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ Circle one : Converges Diverges

13. (a) (5 points) Start from a known series to find a power series about $a = 0$ for $f(x) = \dfrac{1}{4 + x^2}$ and its *radius* of convergence.

(b) (5 points) Use your answer above to find a power series about $a = 0$ for $\dfrac{2x}{(4 + x^2)^2}$ and its *radius* of convergence.

14. (a) (8 points) Use the Maclaurin series for $\sin(t^2)$ to find the Maclaurin series for $\int_0^x \sin(t^2)\, dt$.

(b) (7 points) Use the first 2 terms of the series above to estimate $\int_0^{0.1} \sin(t^2)\, dt$ and estimate the **error**. Do not simplify. Justify the error estimate.

15. (5 points) Determine if $\sum_{n=1}^{\infty} [1 - \cos(1/n)]$ converges or diverges. *Clearly justify your answer!*

HINT: What is the Maclaurin series for $\cos x$?