Name_____ Sec_____

(Print) Last, First Middle

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MATH 152 Final Exam Spring 2002

Sections 513,514 P. Yasskin

1-14	/70
15	/10
16	/10
17	/10

Multiple Choice: (5 points each)

1. $\sum_{n=2}^{\infty} \frac{3^n}{4^{n-1}} =$

a. 4
b. 9
c. $\frac{9}{7}$
d. 3
e. Diverges

2. Find the angle between the vectors $\vec{u} = \langle 1, 1, -1 \rangle$ and $\vec{v} = \langle 1, -2, -1 \rangle$.

a. 0°
b. 30°
c. 45°
d. 60°
e. 90°

3. Find the arc length of the curve $x = 3\cos^2 t \qquad y = 4\sin^2 t$ for $0 \le t \le \frac{\pi}{4}$.

HINT: When you differentiate, remember the chain rule.

a. $\frac{5}{2}$
b. 5
c. 6
d. 12
e. 10π

4. Find an integrating factor for the differential equation $\frac{dy}{dx} = 2xy + \sin x.$

- **a.** $e^{-\cos x}$
- **b.** $e^{-\sin x}$
- **c.** $e^{\cos x}$
- **d.** e^{x^2}
- **e.** e^{-x^2}

5. The area between the curves $y = x^2 + 2$ and $y = 2x + 5$ for $0 \le x \le 6$ is given by the integral:

- **a.** $\int_0^6 (x^2 + 2) - (2x + 5)\,dx$
- **b.** $\int_0^6 (2x + 5) - (x^2 + 2)\,dx$
- **c.** $\int_0^2 (x^2 + 2) - (2x + 5)\,dx + \int_2^6 (2x + 5) - (x^2 + 2)\,dx$
- **d.** $\int_0^2 (2x + 5) - (x^2 + 2)\,dx + \int_2^6 (x^2 + 2) - (2x + 5)\,dx$
- **e.** $\int_0^3 (2x + 5) - (x^2 + 2)\,dx + \int_3^6 (x^2 + 2) - (2x + 5)\,dx$

6. If $\vec{u}$ points North and $\vec{v}$ points South-East, then $\vec{u} \times \vec{v}$ points

- **a.** Up
- **b.** Down
- **c.** East-North-East
- **d.** West-South-West
- **e.** North-West

7. $\int_1^e 9x^2 \ln x\,dx =$

- **a.** $2e^3 + 1$
- **b.** $2e^3 - 2$
- **c.** $2e^3$
- **d.** $3e^3 - 3e^2$
- **e.** $3e^3 - 3e^2 + 3$

8. The region bounded by $x = 0, \quad x = \cos y, \quad y = 0, \quad y = \frac{\pi}{4}$ is rotated about the x-axis. Which integral gives the volume of the solid of revolution?

a. $\int_0^{\pi/4} 2\pi \cos^2 y \, dy$

b. $\int_0^{\sqrt{2}/2} 2\pi x \arccos x \, dx$

c. $\int_0^{\pi/4} 2\pi y \cos y \, dy$

d. $\int_0^{\sqrt{2}/2} \pi(\cos^2 x - x^2) \, dx$

e. $\int_0^{\pi/4} 2\pi y^2 \, dy$

9. With error $|E| < 0.0001$, evaluate $\int_0^{0.1} \sin(x^2) \, dx$. HINT: Use a Maclaurin series.

a. $0.1 - \frac{(0.1)^3}{6}$

b. 0.1

c. $\frac{(0.1)^3}{3}$

d. $(0.1)^2 - \frac{(0.1)^6}{6}$

e. $(0.1)^2$

10. Using a trig substitution, $\int \frac{dx}{\sqrt{9 + 16x^2}}$ becomes

a. $\frac{1}{3} \int \cos\theta \, d\theta$

b. $\frac{1}{3} \int \tan\theta \, d\theta$

c. $\frac{3}{4} \int \sec^2\theta \, d\theta$

d. $\frac{1}{4} \int \sec\theta \, d\theta$

e. $\frac{1}{4} \int \sin\theta \, d\theta$

11. The partial fraction decomposition of $\frac{1}{x^2 - x}$ is

a. $\frac{1}{x-1} + \frac{1}{x}$

b. $\frac{1}{x-1} - \frac{1}{x}$

c. $\frac{1}{x} - \frac{1}{x-1}$

d. $\frac{1}{x} + \frac{1}{x+1}$

e. $\frac{1}{x+1} - \frac{1}{x}$

12. A 4 cm bar has density $\rho = 1 + 5x^3 \ \frac{\text{gm}}{\text{cm}}$ where x is measured from one end. Find its center of mass.

a. $\bar{x} = 324$

b. $\bar{x} = 1032$

c. $\bar{x} = \frac{27}{86}$

d. $\bar{x} = \frac{86}{27}$

e. $\bar{x} = \frac{321}{4}$

13. $\lim_{x \to 0} \frac{\sin x - x \cos x}{x^3} =$

a. $\frac{1}{6}$

b. $\frac{1}{3}$

c. $\frac{1}{2}$

d. $\frac{2}{3}$

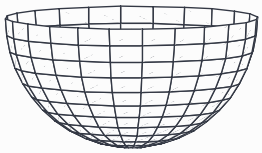
e. ∞

14. If $y(x)$ satisfies the differential equation $\frac{dy}{dx} = \frac{x}{y}$ and the initial condition $y(0) = 3$, find $y(4)$.

a. 1
b. 2
c. 3
d. 4
e. 5

Work Out Problems: (10 points each)

15. Find the work done to pump the water out the top of a hemispherical bowl of radius 5 cm if it is filled to the top. The density of water is $\rho = 1 \frac{\text{gm}}{\text{cm}^3}$. The acceleration of gravity is $g = 980 \frac{\text{cm}}{\text{sec}^2}$.



16. Find the radius and interval of convergence of $\sum_{n=2}^{\infty} \frac{(x-4)^n}{3\ln n}$. Be sure to check the endpoints.

17. Determine if the series $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$ converges absolutely, converges conditionally or diverges. If it converges, find the sum. If it diverges, does it diverge to $+\infty$, $-\infty$ or neither? Show all work and name any tests you use.

Circle One: Converges Absolutely Converges Conditionally Diverges

Fill in the Blank: Converges to _____

Or Circle One: Diverges to $+\infty$ $-\infty$ Neither