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MATH 152 FINAL EXAM Spring 2005

Sections 813-815 Version A1 P. Yasskin

Multiple Choice: (5 points each)

1. Find the average value of $f(x) = \cos x$ on the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

a. $\frac{2\sqrt{2}}{\pi}$

b. $\frac{\sqrt{2}}{\pi}$

c. $\sqrt{2}$

d. $\frac{1}{\sqrt{2}}$

e. $\frac{\pi}{\sqrt{2}}$

2. The ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$ is revolved about the x-axis. Which integral gives the volume of the resulting ellipsoid?

a. $\int_{-2}^{2} 2\pi x\sqrt{16 - 4x^2}\, dx$

b. $\int_{-4}^{4} 2\pi(16 - 4x^2)^2\, dx$

c. $\int_{-2}^{2} \pi(16 - 4x^2)\, dx$

d. $\int_{-4}^{4} 2\pi x\sqrt{16 - 4x^2}\, dx$

e. $\int_{-2}^{2} \pi(16 - 4x^2)^2\, dx$

3. Compute $\int_0^{\pi/4} \cos\theta \sin^3\theta\, d\theta.$

a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{1}{8}$

d. $\frac{1}{16}$

e. $\frac{1}{32}$

4. Compute $\int_0^{\ln 2} xe^{-x}\,dx.$

a. $\frac{1}{2} + \frac{1}{2}\ln 2$

b. $\frac{1}{2} - \frac{1}{2}\ln 2$

c. $\frac{1}{2}\ln 2 - \frac{1}{2}$

d. $-\frac{1}{2}\ln 2 - \frac{1}{2}$

e. Divergent

5. Use the Trapezoid Rule with $n = 4$ intervals to approximate the integral $\int_1^9 (9 + x^2)\,dx.$

a. 240

b. 312

c. $314\frac{1}{3}$

d. 320

e. 400

6. A barrel initially contains 3 cups of sugar dissolved in 4 gallons of water. You then add pure water at the rate of 2 gallons per minute while the mixture is draining out of a hole in the bottom at 2 gallons per minute. Find the amount of sugar in the barrel after 2 minute.

a. $\frac{3}{\sqrt{e}}$

b. $\frac{3}{e}$

c. $3e$

d. $3\sqrt{e}$

e. $\frac{3}{e^2}$

7. As n approaches infinity, the sequence $\left\{\frac{1-\cos n}{n^2}\right\}$

a. converges to $-\frac{1}{2}$

b. converges to 0

c. converges to $\frac{1}{2}$

d. converges to 1

e. diverges

8. Compute $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}-\frac{n+1}{n+2}\right)$

a. $-\frac{1}{2}$

b. $\frac{1}{2}$

c. 1

d. 2

e. Divergent

9. Find the 4^{th} degree Taylor polynomial for $f(x)=x^2-x$ about $x=2$.

a. $T_4(x)=2+3(x-2)+(x-2)^2+3(x-2)^3+(x-2)^4$

b. $T_4(x)=2+3(x-2)+2(x-2)^2+3(x-2)^3+2(x-2)^4$

c. $T_4(x)=2+3(x-2)+(x-2)^2$

d. $T_4(x)=2+3(x-2)+2(x-2)^2$

e. $T_4(x)$ cannot be found because $x=2$ is outside the interval of convergence.

10. A triangle has vertices $A = (0,3,2)$, $B = (-2,3,0)$ and $C = (-2,0,3)$. Find the angle at vertex B.

a. $\frac{\pi}{6}$

b. $\frac{\pi}{3}$

c. $\frac{\pi}{2}$

d. $\frac{2\pi}{3}$

e. $\frac{5\pi}{6}$

11. If $\vec{u}$ points South-West and $\vec{v}$ points Up, which way does $\vec{u} \times \vec{v}$ point?

a. South-East

b. North-East

c. North-West

d. $45°$ Up from North-West

e. $45°$ Down from North-West

12. Find the area of a triangle with edges $\vec{a} = (3,-2,1)$ and $\vec{b} = (-1,0,1)$.

a. 1

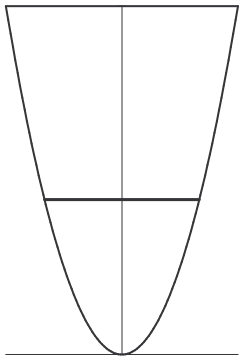
b. 2

c. $\sqrt{6}$

d. 6

e. $2\sqrt{6}$

13. (12 points) The end of a water trough occupies the region between $y = x^2$ m and $y = 9$ m.
It is filled to a depth of $y = 4$ m.
Find the force on the end of the trough.
Give your answer in terms of ρ (the density of water) and g (the acceleration of gravity).



14. (12 points) Compute $\int_3^{3\sqrt{2}} \frac{\sqrt{x^2-9}}{x}\,dx$.

15. (12 points) Find the arc length of the curve $y = \frac{x^2}{4} - \frac{\ln x}{2}$ between $x = 1$ and $x = e$.

16. (12 points) The Taylor series $f(x) = \sum_{n=1}^{\infty} \frac{n}{2^n}(x-1)^{n-1}$ is obtained by differentiating the series $g(x) = \sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n} = \sum_{n=0}^{\infty} \left(\frac{x-1}{2}\right)^n$. What is the the function $f(x)$? What is the interval of convergence for $f(x)$ (including endpoints)? Justify your answers.