MATH 152, SPRING 2012
COMMON EXAM II - VERSION A

Last Name: _____ First Name: _____

Signature: _____ Section No: _____

PART I: Multiple Choice (4 pts each)

1. $\int \dfrac{1}{x^2(x-1)}\,dx =$

    (a) $\ln|x-1| + \dfrac{1}{x} + C$

    (b) $\ln|x^2(x-1)| + C$

    (c) $\ln|x| - \dfrac{1}{x} - \ln|x-1| + C$

    (d) $-\ln|x| + \dfrac{1}{x} + \ln|x-1| + C$

    (e) $\ln|x-1| - \dfrac{1}{x} + C$

2. The improper integral $\displaystyle\int_1^e \dfrac{dx}{x\ln x}$

    (a) diverges to ∞.

    (b) diverges to $-\infty$.

    (c) converges to 1.

    (d) converges to -1.

    (e) converges to $\dfrac{1}{e} - 1$.

3. Which of the following integrals gives the surface area obtained by rotating the curve $y = e^{-4x}$, for $0 \le x \le 1$, about the y-axis?

    (a) $\displaystyle\int_0^1 2\pi e^{-4x}\sqrt{1 + 16e^{-8x}}\,dx$

    (b) $\displaystyle\int_0^1 2\pi x\sqrt{1 + 16e^{-8x}}\,dx$

    (c) $\displaystyle\int_1^{e^{-4}} 2\pi y\sqrt{1 + \dfrac{1}{16y^2}}\,dy$

    (d) $\displaystyle\int_0^1 \dfrac{\pi}{2}\sqrt{16y^2 + 1}\,dy$

    (e) $\displaystyle\int_0^1 \dfrac{\pi}{8}\dfrac{\ln y}{y}\sqrt{16y^2 + 1}\,dy$

1

4. Compute $\displaystyle\int_{-1}^{\infty} \frac{dx}{1+x^2}$.

    (a) $\dfrac{3\pi}{4}$

    (b) $\dfrac{\pi}{2}$

    (c) $\dfrac{\pi}{4}$

    (d) ∞

    (e) 0

5. By substituting $x = 3\tan\theta$, the integral $\displaystyle\int_0^3 x^2\sqrt{x^2+9}\,dx$ becomes

    (a) $\displaystyle\int_0^{\pi/4} 27\tan^2\theta\sec\theta\,d\theta$

    (b) $\displaystyle\int_0^3 27\tan^2\theta\sec^3\theta\,d\theta$

    (c) $\displaystyle\int_0^{\pi/4} 81\tan^3\theta\sec^2\theta\,d\theta$

    (d) $\displaystyle\int_0^{\pi/4} 81\tan^2\theta\sec^2\theta\,d\theta$

    (e) $\displaystyle\int_0^{\pi/4} 81\tan^2\theta\sec^3\theta\,d\theta$

6. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n + 2^n}{6^n} =$

    (a) $\dfrac{1}{3}$

    (b) $\dfrac{5}{14}$

    (c) $\dfrac{33}{14}$

    (d) $\dfrac{3}{10}$

    (e) $\dfrac{27}{10}$

7. Find the length of the curve $x = t^2$, $y = t^3$, for $0 \leq t \leq 1$.

(a) $\dfrac{1}{27}\left(13\sqrt{13} - 8\right)$

(b) $\dfrac{2\pi}{27}\left(13\sqrt{13} - 8\right)$

(c) $\dfrac{1}{27}\left(13\sqrt{13} - 1\right)$

(d) $\dfrac{1}{27}$

(e) $\dfrac{2\pi}{27}\left(13\sqrt{13} - 1\right)$

8. Which of the following series diverges by the Test for Divergence?

(a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\ln n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\dfrac{\pi}{2} - \dfrac{1}{n}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\dfrac{1}{n}\right)$

(e) The Test for Divergence is inconclusive for all of the above series.

9. The recursive sequence defined by $a_1 = 2$, $a_{n+1} = 5 - \dfrac{4}{a_n}$ converges. Find the limit.

(a) 1

(b) 4

(c) 5

(d) $\dfrac{5}{2}$

(e) 2

10. Which of the following statements is true regarding the improper integral $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x + \sqrt{x}}$?

(a) The integral converges because $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x + \sqrt{x}} < \int_1^{\infty} \dfrac{dx}{\sqrt{x}}$ and $\displaystyle\int_1^{\infty} \dfrac{dx}{\sqrt{x}}$ converges.

(b) The integral diverges because $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x + \sqrt{x}} > \int_1^{\infty} \dfrac{dx}{\sqrt{x}}$ and $\displaystyle\int_1^{\infty} \dfrac{dx}{\sqrt{x}}$ diverges.

(c) The integral diverges because $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x + \sqrt{x}} > \int_1^{\infty} \dfrac{dx}{e^x}$ and $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x}$ diverges.

(d) The integral converges because $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x + \sqrt{x}} < \int_1^{\infty} \dfrac{dx}{e^x}$ and $\displaystyle\int_1^{\infty} \dfrac{dx}{e^x}$ converges.

(e) The integral converges to 0.

11. The sequence whose terms are $a_n = \dfrac{n^2 - 1}{n^2}$

    (a) decreases and converges to 1.

    (b) increases and converges to 1.

    (c) decreases and converges to 0.

    (d) increases and converges to 0.

    (e) diverges.

12. Find the surface area obtained by rotating the curve $x = \cos(2t)$, $y = \sin(2t)$, for $0 \le t \le \dfrac{\pi}{4}$, about the x-axis.

    (a) $\dfrac{\pi}{4}$

    (b) 2π

    (c) $\dfrac{\pi}{2}$

    (d) π

    (e) 4π

13. Find the sum of the geometric series $S = \dfrac{4}{9} + \dfrac{8}{27} + \dfrac{16}{81} + \dots.$

    (a) $S = 3$

    (b) $S = 2$

    (c) $S = \dfrac{2}{3}$

    (d) $S = \dfrac{4}{15}$

    (e) $S = \dfrac{4}{3}$

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

14. (10 pts) Integrate $\displaystyle\int \sqrt{16 - 9x^2}\, dx$.

15. (8 pts) Find the sum of the series: $\displaystyle S = \sum_{n=1}^{\infty} \left(\cos\frac{\pi}{n} - \cos\frac{\pi}{n+1} \right)$

16. (10 pts) Integrate $\displaystyle\int \frac{4x^2 - 1}{(x^2 + 1)(x - 2)}\, dx.$

17. If the nth partial sum of the series $\sum\limits_{n=1}^{\infty} a_n$ is given by $s_n = \dfrac{2n+1}{n}$,

   (i) (5 pts) Find a_{10}.

   (ii) (5 pts) Find the sum of the series $S = \sum\limits_{n=1}^{\infty} a_n$.

18. (10 pts) Find the surface area obtained by rotating the curve $y = \dfrac{x^2}{4} - \dfrac{1}{2}\ln x$, for $1 \leq x \leq 2$, about the y-axis.

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Question	Points Awarded	Points
1-13		52
14		10
15		8
16		10
17		10
18		10
		100