MATH 152, SPRING 2012
COMMON EXAM II - VERSION B

Last Name: _____ **First Name**: _____

Signature: _____ **Section No**: _____

PART I: Multiple Choice (4 pts each)

1. The sequence whose terms are $a_n = \dfrac{n^2-1}{n^2}$

   (a) increases and converges to 1.
   (b) decreases and converges to 1.
   (c) increases and converges to 0.
   (d) decreases and converges to 0.
   (e) diverges.

2. By substituting $x = 3\tan\theta$, the integral $\displaystyle\int_0^3 x^2\sqrt{x^2+9}\,dx$ becomes

   (a) $\displaystyle\int_0^{\pi/4} 81\tan^2\theta\sec^2\theta\,d\theta$
   (b) $\displaystyle\int_0^{\pi/4} 81\tan^3\theta\sec^2\theta\,d\theta$
   (c) $\displaystyle\int_0^{\pi/4} 27\tan^2\theta\sec\theta\,d\theta$
   (d) $\displaystyle\int_0^{\pi/4} 81\tan^2\theta\sec^3\theta\,d\theta$
   (e) $\displaystyle\int_0^{3} 27\tan^2\theta\sec^3\theta\,d\theta$

3. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n+2^n}{6^n} =$

   (a) $\dfrac{5}{14}$
   (b) $\dfrac{27}{10}$
   (c) $\dfrac{3}{10}$
   (d) $\dfrac{33}{14}$
   (e) $\dfrac{1}{3}$

4. Which of the following series diverges by the Test for Divergence?

   (a) $\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{2} - \frac{1}{n}\right)$

   (b) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

   (c) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

   (d) $\sum_{n=1}^{\infty} \frac{n}{n!}$

   (e) The Test for Divergence is inconclusive for all of the above series.

5. Find the length of the curve $x = t^2$, $y = t^3$, for $0 \leq t \leq 1$.

   (a) $\frac{2\pi}{27}\left(13\sqrt{13} - 1\right)$

   (b) $\frac{1}{27}$

   (c) $\frac{1}{27}\left(13\sqrt{13} - 1\right)$

   (d) $\frac{2\pi}{27}\left(13\sqrt{13} - 8\right)$

   (e) $\frac{1}{27}\left(13\sqrt{13} - 8\right)$

6. Find the surface area obtained by rotating the curve $x = \cos(2t)$, $y = \sin(2t)$, for $0 \leq t \leq \frac{\pi}{4}$, about the x-axis.

   (a) $\frac{\pi}{2}$

   (b) π

   (c) $\frac{\pi}{4}$

   (d) 4π

   (e) 2π

7. Find the sum of the geometric series $S = \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + ...$

   (a) $S = 2$

   (b) $S = 3$

   (c) $S = \frac{4}{3}$

   (d) $S = \frac{4}{15}$

   (e) $S = \frac{2}{3}$

8. Which of the following statements is true regarding the improper integral $\displaystyle\int_1^\infty \frac{dx}{e^x+\sqrt{x}}$?

(a) The integral converges to 0.

(b) The integral converges because $\displaystyle\int_1^\infty \frac{dx}{e^x+\sqrt{x}} < \int_1^\infty \frac{dx}{e^x}$ and $\displaystyle\int_1^\infty \frac{dx}{e^x}$ converges.

(c) The integral diverges because $\displaystyle\int_1^\infty \frac{dx}{e^x+\sqrt{x}} > \int_1^\infty \frac{dx}{\sqrt{x}}$ and $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}}$ diverges.

(d) The integral diverges because $\displaystyle\int_1^\infty \frac{dx}{e^x+\sqrt{x}} > \int_1^\infty \frac{dx}{e^x}$ and $\displaystyle\int_1^\infty \frac{dx}{e^x}$ diverges.

(e) The integral converges because $\displaystyle\int_1^\infty \frac{dx}{e^x+\sqrt{x}} < \int_1^\infty \frac{dx}{\sqrt{x}}$ and $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}}$ converges.

9. The recursive sequence defined by $a_1 = 2$, $a_{n+1} = 5 - \dfrac{4}{a_n}$ converges. Find the limit.

(a) 4

(b) 5

(c) 1

(d) $\dfrac{5}{2}$

(e) 2

10. $\displaystyle\int \frac{1}{x^2(x-1)}\,dx =$

(a) $\ln|x-1| - \dfrac{1}{x} + C$

(b) $\ln|x| - \dfrac{1}{x} - \ln|x-1| + C$

(c) $-\ln|x| + \dfrac{1}{x} + \ln|x-1| + C$

(d) $\ln|x-1| + \dfrac{1}{x} + C$

(e) $\ln|x^2(x-1)| + C$

11. Compute $\displaystyle\int_{-1}^{\infty} \frac{dx}{1+x^2}$.

(a) $\dfrac{\pi}{4}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{3\pi}{4}$

(d) ∞

(e) 0

12. Which of the following integrals gives the surface area obtained by rotating the curve $y = e^{-4x}$, for $0 \le x \le 1$, about the y-axis?

(a) $\displaystyle\int_0^1 2\pi x\sqrt{1+16e^{-8x}}\,dx$

(b) $\displaystyle\int_0^1 2\pi e^{-4x}\sqrt{1+16e^{-8x}}\,dx$

(c) $\displaystyle\int_1^{e^{-4}} 2\pi y\sqrt{1+\frac{1}{16y^2}}\,dy$

(d) $\displaystyle\int_0^1 \frac{\pi}{2}\sqrt{16y^2+1}\,dy$

(e) $\displaystyle\int_0^1 \frac{\pi}{8}\frac{\ln y}{y}\sqrt{16y^2+1}\,dy$

13. The improper integral $\displaystyle\int_1^e \frac{dx}{x\ln x}$

(a) diverges to $-\infty$.

(b) diverges to ∞.

(c) converges to -1

(d) converges to 1.

(e) converges to $\dfrac{1}{e}-1$.

PART II: WORK OUT (52 points total)

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

14. (10 pts) Integrate $\int \sqrt{16 - 9x^2}\, dx$.

15. (8 pts) Find the sum of the series: $S = \sum_{n=1}^{\infty} \left(\cos \frac{\pi}{n} - \cos \frac{\pi}{n+1} \right)$.

16. (10 pts) Integrate $\displaystyle\int \frac{4x^2 - 1}{(x^2 + 1)(x - 2)}\, dx$.

17. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is given by $s_n = \dfrac{2n+1}{n}$,

(i) (5 pts) Find a_{10}.

(ii) (5 pts) Find the sum of the series $S = \sum_{n=1}^{\infty} a_n$.

18. (10 pts) Find the surface area obtained by rotating the curve $y = \dfrac{x^2}{4} - \dfrac{1}{2}\ln x$, for $1 \leq x \leq 2$, about the y-axis.

Last Name: _____ **First Name**: _____

Section No: _____

MATH 152, SPRING 2012
COMMON EXAM II - VERSION B

Question	Points Awarded	Points
1-13		52
14		10
15		8
16		10
17		10
18		10
		100