Name______________________________ Section____________

MATH 152 FINAL EXAM Spring 2014

Sections 549-551 P. Yasskin

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15	/15
16	/20
17	/10
18	/5
Total	/106

Multiple Choice: (14 problems, 4 points each)

1. Find the area between the curves $y = 2x^2$ and $y = 4x$.

a. $\frac{4}{3}$

b. $\frac{8}{3}$

c. $\frac{16}{3}$

d. $\frac{32}{3}$

e. $\frac{64}{3}$

2. Find the average value of the function $f(x) = e^x$ on the interval $[-1,1]$.

a. $e + \frac{1}{e}$

b. $\frac{1}{2}\left(e + \frac{1}{e}\right)$

c. $e - \frac{1}{e}$

d. $\frac{1}{2}\left(e - \frac{1}{e}\right)$

e. 1

3. Compute $\int_{-\pi/2}^{\pi/2} \sin^4\theta \cos\theta \, d\theta$.

- **a.** -6
- **b.** $-\frac{2}{5}$
- **c.** $\frac{2}{5}$
- **d.** $\frac{2}{3}$
- **e.** 6

4. Compute $\int x^2 e^{2x} \, dx$.

- **a.** $\left(\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4}\right)e^{2x} + C$
- **b.** $\left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right)e^{2x} + C$
- **c.** $\left(\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{4}\right)e^{2x} + C$
- **d.** $\left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2}\right)e^{2x} + C$
- **e.** $\left(\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{2}\right)e^{2x} + C$

5. Compute $\int \frac{\sqrt{x^2 - 1}}{x} \, dx$. HINT: $\tan^2\theta = \sec^2\theta - 1$

- **a.** $\operatorname{arcsec} x - \sqrt{x^2 - 1} + C$
- **b.** $\sqrt{x^2 - 1} + \operatorname{arcsec} x + C$
- **c.** $\sqrt{x^2 - 1} - \operatorname{arcsec} x + C$
- **d.** $\frac{1}{3}(x^2 - 1)^{3/2} - (x^2 - 1)^{1/2} + C$
- **e.** $\frac{1}{3}(x^2 - 1)^{3/2} + (x^2 - 1)^{1/2} + C$

6. In the partial fraction expansion, $\frac{4x^5+2x^2+8}{x^3(x^2+4)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$, find C.

a. $\frac{1}{4}$

b. $\frac{1}{2}$

c. 2

d. 8

e. 16

7. Compute $\int_{-2}^{2} \frac{1}{x^3}\,dx$.

a. $-\frac{1}{4}$

b. $-\frac{1}{8}$

c. 0

d. $\frac{1}{4}$

e. undefined

8. The region under the curve $y = \frac{1}{x^2+1}$ above the interval $[0,1]$ is revolved about the y-axis. Find the volume of the resulting solid.

a. $\pi \ln(2)$

b. $\pi \ln(2) - \pi$

c. $2\pi \ln(2)$

d. $4\pi \ln(2)$

e. $4\pi \ln(2) - 4\pi$

9. Find the arclength of the curve $x = 2t^4$, $y = t^6$ between $t = 0$ and $t = 1$.

a. $\dfrac{1}{27}$

b. $\dfrac{1}{3}$

c. $\dfrac{61}{12}$

d. $\dfrac{61}{27}$

e. $\dfrac{61}{54}$

10. The curve $x = 2t^4$, $y = t^6$ between $t = 0$ and $t = 1$ is rotated about the x-axis. Which integral gives the area of the resulting surface?

a. $\int_0^1 4\pi t^9 \sqrt{16 + 9t^4}\, dt$

b. $\int_0^1 8\pi t^7 \sqrt{16 + 9t^4}\, dt$

c. $\int_0^1 4\pi t^7 \sqrt{16 + 9t^4}\, dt$

d. $\int_0^1 2\pi t^7 \sqrt{16 + 9t^4}\, dt$

e. $\int_0^1 4\pi t^3 \sqrt{16 + 9t^4}\, dt$

11. The series $S = \sum_{n=0}^{\infty} \dfrac{(-1)^n 3^n}{2^{2n}}$

a. converges to 4

b. converges to $\dfrac{4}{3}$

c. converges to $\dfrac{4}{7}$

d. converges to $\dfrac{2}{5}$

e. diverges

12. Compute $\lim_{x \to 0} \frac{6x - x^3 - 6\sin x}{x^5}$

a. $\frac{6}{5!}$

b. -6

c. 6

d. $\frac{2}{3!}$

e. $-\frac{1}{20}$

13. Compute $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n)!}$

a. $\frac{1}{2}$

b. -1

c. 0

d. 1

e. $\frac{1}{\sqrt{2}}$

14. Find the center and radius of the sphere $x^2 + 4x + y^2 + z^2 - 6z + 4 = 0$

a. center: $(-2, 0, 3)$ radius: $R = 2$

b. center: $(2, 0, -3)$ radius: $R = 9$

c. center: $(-2, 0, 3)$ radius: $R = 9$

d. center: $(2, 0, -3)$ radius: $R = 3$

e. center: $(-2, 0, 3)$ radius: $R = 3$

Work Out (4 questions, Points indicated)

Show all you work.

15. (15 points) Consider the series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2^{n-1}}$.

a. (3) Show the series is convergent.

b. (6 Extra Credit) Show the series is absolutely convergent.
HINT: You may use these formulas without proof, if neeeded:
$2^{n-1} > n^3$ for $n \geq 12$ $\qquad \int b^x\, dx = \frac{b^x}{\ln b} + C$ $\qquad \int x\, b^x\, dx = \frac{x b^x}{\ln b} - \frac{b^x}{\ln^2 b} + C$

c. (3) Compute S_7, the 7^{th} partial sum for S. Do not simplify.

d. (3) Find a bound on the remainder $|R_7| = |S - S_7|$ when S_7 is used to approximate S. Name the theorem you used.

16. (20 points) Let $f(x) = \ln(x)$.

a. (6) Find the Taylor series for $f(x)$ centered at $x = 3$.

b. (11) The Taylor series for $f(x)$ centered at $x = 4$ is $T(x) = 2\ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4^n n}(x-4)^n$.

Find the interval of convergence for the Taylor series centered at $x = 4$. Give the radius and check the endpoints.

c. (3) If the cubic Taylor polynomial centered at $x = 4$ is used to approximate $\ln(x)$ on the interval $[3, 6]$, use the Taylor's Inequality to bound the error.

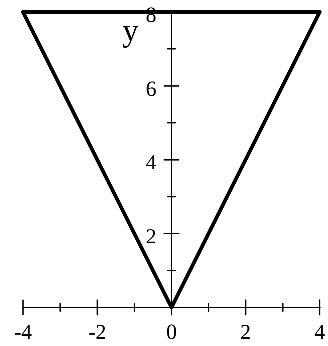
Taylor's Inequality:

Let $T_n(x)$ be the n^{th}-degree Taylor polynomial for $f(x)$ centered at $x = a$ and let $R_n(x) = f(x) - T_n(x)$ be the remainder. Then

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$$

provided $M \geq |f^{(n+1)}(c)|$ for all c between a and x.

17. $\left(10\text{ points}\right)$ A conical tank with vertex down, radius 4, and height 8, is filled with water.
Find the work done to pump the water out the top of the tank.
Take the density of water to be ρ
and the acceleration of gravity to be g.



18. $\left(5\text{ points}\right)$ The region between the curves $y = 2x^2$ and $y = 4x$ is rotated about the x-axis.
Find the volume of the resulting solid.