

Name \_\_\_\_\_ Section \_\_\_\_\_

MATH 152

FINAL EXAM Version A

Spring 2016

Sections 555-557

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1-13	/52
14	/20
15	/20
16	/5
17	/5
18	/5
Total	/107

Multiple Choice: (13 problems, 4 points each)

1.

**Average Value of a Function**

New Problem or Modify or Make Your Own Problem

Find the average value of the function  $f(x) = \sin(x)$  on the interval  $[a,b] = [0,\pi]$ .

- a.  $\frac{2}{\pi}$
- b.  $\frac{1}{\pi}$
- c.  $2\pi$
- d. 2
- e. 1

2.

**Integrals Which are Improper at an Endpoint**

New Problem

Problem Statement:

Determine if the following improper integral is convergent or divergent.

$$\int_{-2}^{\infty} (x+4)^{-\frac{1}{3}} dx$$

If convergent, compute it.  
If divergent, determine if it is +infinity, -infinity, or neither.

- a. converges to  $\frac{3}{2^{1/3}}$
- b. converges to  $-\frac{3}{2^{1/3}}$
- c. diverges to  $\infty$
- d. diverges to  $-\infty$
- e. diverges but not to  $\pm\infty$

3.

**Integration By Parts**

Use integration by parts  
 to compute the integral:

$$J = \int_1^2 \frac{\ln(x)}{x^2} dx$$

- a.  $\frac{3 - \ln(2)}{2}$
- b.  $\frac{\ln(2) - 3}{2}$
- c.  $\frac{\ln(2) - 1}{2}$
- d.  $\frac{-\ln(2)}{2}$
- e.  $\frac{1 - \ln(2)}{2}$

4.

**Trigonometric Integrals**

Use a substitution  
 to compute the integral:

$$J = \int_0^{\frac{1}{2}\pi} \sin^3 x dx$$

- a.  $-\frac{1}{4}$
- b.  $\frac{1}{4}$
- c.  $\frac{2}{3}$
- d.  $-\frac{4}{3}$
- e.  $\frac{4}{3}$

5.

**Integration by Trigonometric Substitution**

New Integral

Goal: Evaluate the indefinite integral using a trigonometric substitution:

$$I = \int (x^2 + 16)^{-\frac{3}{2}} dx$$

Simply identify the integral after the substitution.

- a.  $\frac{1}{16} \int \csc^2 \theta d\theta$
- b.  $\frac{1}{64} \int \sec^2 \theta d\theta$
- c.  $\frac{1}{16} \int \sin^3 \theta d\theta$
- d.  $\frac{1}{16} \int \cos \theta d\theta$
- e.  $\frac{1}{64} \int \cos^3 \theta d\theta$

6.

**Partial Fractions: Finding Coefficients**

New Function  Include Completing the Square

Goal: Find the coefficients in the partial fraction expansion:

$$\frac{-2x^2 - x + 2}{x^2(x-1)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x-1}$$

Just find  $A_1$  and  $A_2$ .

- a.  $A_1 = -1 \quad A_2 = -2$
- b.  $A_1 = 1 \quad A_2 = 2$
- c.  $A_1 = -2 \quad A_2 = -1$
- d.  $A_1 = 2 \quad A_2 = 1$
- e.  $A_1 = -2 \quad A_2 = 1$

7.

**Volume Of Revolution** [ - ] [ □ ] [ × ]

New Problem or Modify or Make Your Own Problem [ Quit ]

The region to the right of  $x = 2y^2$ , to the left of  $x = 4y$ , and between  $y = 0$  and  $y = 2$  is rotated about the  $x$ -axis. Find the volume swept out.

- a.  $\frac{8}{3}\pi$
- b.  $\frac{16}{3}\pi$
- c.  $\frac{256}{3}\pi$
- d.  $\frac{16}{15}\pi$
- e.  $\frac{256}{15}\pi$

8.

**Volume Of Revolution** [ - ] [ □ ] [ × ]

New Problem or Modify or Make Your Own Problem [ Quit ]

The region to the right of  $x = 2y^2$ , to the left of  $x = 4y$ , and between  $y = 0$  and  $y = 2$  is rotated about the  $y$ -axis. Find the volume swept out.

- a.  $\frac{8}{3}\pi$
- b.  $\frac{16}{3}\pi$
- c.  $\frac{256}{3}\pi$
- d.  $\frac{16}{15}\pi$
- e.  $\frac{256}{15}\pi$

9.

**Surface Area Of Solid Of Revolution**

New Problem

or

Modify or Make Your Own Problem

The curve  $y = \frac{2}{3}x^2$ , between  $x = 0$  and  $x = 1$ , is rotated about the  $y$ -axis. Find the surface area of the surface of revolution.

- a.  $\frac{49}{36}$
- b.  $\frac{49}{72}$
- c.  $\frac{49}{144}$
- d.  $\frac{49}{36}\pi$
- e.  $\frac{126}{72}\pi$

10.

**Work to Lift an Object with a Rope**

New Problem

Goal:

Find the work needed to lift a 12 lb object up a 50 ft building using a rope whose density is 4 lb/ft.

- a. 5600 ft-lb
- b. 5000 ft-lb
- c. 3100 ft-lb
- d. 2500 ft-lb
- e. 600 ft-lb

11. **Geometric Series**

New Series

Goal: Compute the sum of the geometric series (if the sum exists).

S =  $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} 3 \left(-\frac{1}{3}\right)^n$

- a.  $-\frac{3}{2}$
- b.  $-\frac{3}{4}$
- c.  $\frac{9}{4}$
- d.  $\frac{9}{2}$
- e. diverges

12. **Computing Limits Using Maclaurin Series**

New Limit

Goal: Use a Maclaurin Series to evaluate this limit:

$L = \lim_{x \rightarrow 0} \frac{e^{(x^3)} - x^3 - 1}{x^6}$

- a.  $\frac{1}{2}$
- b.  $\frac{1}{3}$
- c.  $\frac{1}{6}$
- d.  $\frac{1}{6!}$
- e. diverges

13. **Triangles: Angles in 3D**

New Problem   Radians

Goal: Find the angle at A in the triangle with vertices:

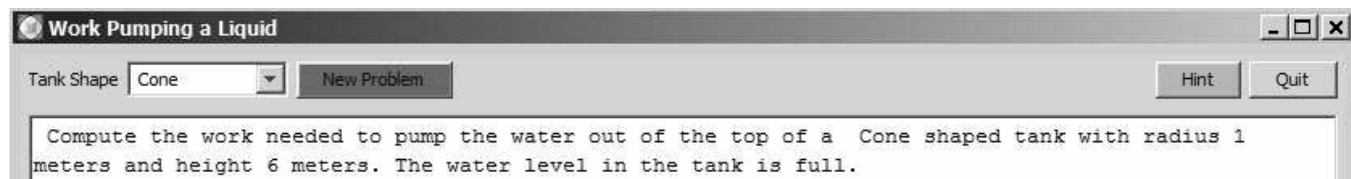
A = [1, 0, 2]      B = [-2, -6, -4]

C = [3, -2, 3]

- a. 0
- b.  $\frac{\pi}{2}$
- c.  $\arccos\left(\frac{2}{3}\right)$
- d.  $\arccos\left(\frac{2}{9}\right)$
- e.  $\arccos\left(\frac{4}{9}\right)$

Work Out (5 questions, Points indicated. Show all you work.)

14. (20 points)



Write your answer as a multiple of  $\rho g$  where  $\rho$  is the density of water and  $g$  is the acceleration of gravity. The vertex of the cone is at the bottom.

15. (20 points)

Center And Radius of Convergence of a Power Series

Goal: Find the center and radius of convergence of the series:

New Series

$$\sum_{n=1}^{\infty} \frac{n^5 (x-6)^n}{5^n}$$

Also find the interval of convergence by checking the endpoints.

a. (2 pts) Identify the center:

$a =$  \_\_\_\_\_

b. (8 pts) Find the radius of convergence:

$R =$  \_\_\_\_\_

c. (8 pts) Check the endpoints:

d. (2 pts) Summarize the interval of convergence:

$I =$  \_\_\_\_\_



16. (5 points) Determine whether the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$  is absolutely convergent, convergent but not absolutely or divergent. Explain all tests you use.

17. (5 points) The series  $S = \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$  converges by the Integral Test.

If it is approximated by its 100<sup>th</sup> partial sum  $S_{100}$ , compute the integral bound on the error in this approximation.

18. (5 points) Compute the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)! 3^{2n+1}}$ .