MATH 152 Spring 2016
COMMON EXAM II - VERSION B

LAST NAME: Key FIRST NAME: ____________

INSTRUCTOR: ____________

SECTION NUMBER: ____________

UIN: ____________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		5
17		8
18		12
19		10
20		9
21		11
Total		100

PART I: Multiple Choice. 3 points each.

1. Which of the following is the correct partial fraction decomposition for $f(x) = \dfrac{4x+3}{x^2(x^2-9)(x^2+4)}$?

(a) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx}{x^2-9} + \dfrac{Ex}{x^2+4}$

(b) $\dfrac{A}{x^2} + \dfrac{Bx+C}{x^2-9} + \dfrac{Dx+E}{x^2+4}$

(c) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{x-3} + \dfrac{E}{x+2} + \dfrac{F}{(x+2)^2}$

(d) $\dfrac{A}{x^2} + \dfrac{B}{x+3} + \dfrac{C}{x-3} + \dfrac{Dx+E}{x^2+4}$

(e) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{x-3} + \dfrac{Ex+F}{x^2+4}$

$$\frac{4x+3}{x^2(x+3)(x-3)(x^2+4)}$$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{5^{n+1}} =$

$$\sum_{n=0}^{\infty} \frac{4^n}{5^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{5}\left(\frac{4}{5}\right)^n$$

(a) 1

(b) $\dfrac{1}{9}$

(c) 4

(d) $\dfrac{1}{3}$

(e) $\dfrac{4}{3}$

$$= \frac{\frac{1}{5}}{1-\frac{4}{5}} \frac{5}{5}$$

$$= \frac{1}{5-4}$$

$$= 1$$

3. Find the length of the curve $x = \dfrac{t^2}{2}$, $y = \dfrac{t^3}{3}$, $0 \le t \le 1$.

(a) $\dfrac{4}{3}\left(2^{3/2}-1\right)$

(b) $\dfrac{1}{3}\left(2^{3/2}-1\right)$

(c) $\dfrac{1}{24}\left(10^{3/2}-1\right)$

(d) $\dfrac{1}{54}\left(10^{3/2}-1\right)$

(e) $3(2^{3/2}-1)$

$$L = \int_0^1 \sqrt{t^2+t^4}\,dt$$

$$= \int_0^1 t\sqrt{1+t^2}\,dt = \frac{1}{2}\frac{2}{3}(1+t^2)^{3/2}\Big|_0^1$$

$$= \frac{1}{3}\left(2^{3/2}-1\right)$$

4. The integral $\int_1^\infty \frac{dx}{\sqrt{x}+e^{9x}}$

(a) diverges by comparison with $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

(b) converges to 0

(c) converges by comparison with $\int_1^\infty \frac{1}{e^{9x}}\,dx$

(d) converges by comparison with $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

(e) diverges by comparison with $\int_1^\infty \frac{1}{e^{9x}}\,dx$

$$0 < \int_1^\infty \frac{dx}{\sqrt{x}+e^{9x}} < \int_1^\infty \frac{dx}{e^{9x}}$$

$$\lim_{t\to\infty} \int_1^t e^{-9x}\,dx$$

$$\lim_{t\to\infty} -\frac{1}{9}e^{-9x}\Big|_1^t$$

$$\lim_{t\to\infty} -\frac{1}{9}\left(e^{-9t}-e^{-9}\right)$$

$$= \frac{1}{9e^9}$$

larger integral converges, so does smaller by Comparison Test

5. Given the sequence $a_1 = 1$ and $a_{n+1} = \sqrt{12+a_n}$ is increasing and bounded, what statement is true about a_n?

(a) converges to 6

(b) diverges

(c) converges to 2

(d) converges to 3

(e) converges to 4

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{12+a_n}$$

let $\lim_{n\to\infty} a_n = L$

Then $\lim_{n\to\infty} a_{n+1} = L$

$$L = \sqrt{12+L}$$

$$L^2 = 12+L$$

$$L^2 - L - 12 = 0$$

$$(L-4)(L+3) = 0$$

since $a_1 = 1$ and a_n is increasing

$\boxed{L=4}$

$L = -3$

6. Given the sequence $a_n = \frac{\ln n}{n}$, $n \geq 5$, which of the following statements are true?

I. a_n is decreasing II. $(-1)^n a_n$ converges to 0 III. a_n is bounded

(a) only I. and II.

(b) only I. and III.

(c) only II.

(d) only II. and III.

(e) All of the above statements are true.

$$a_n = \frac{\ln n}{n} = \left\{\frac{\ln 5}{5}, \frac{\ln 6}{6}, \dots\right\}$$

since $\frac{d}{dx}\frac{\ln x}{x} = \frac{1-\ln x}{x^2} < 0$

$\left\{\frac{\ln n}{n}\right\}$ is decreasing

since $\lim_{n\to\infty} a_n = 0$

$$\lim_{n\to\infty} (-1)^n a_n = 0$$

$$\lim_{n\to\infty} \frac{\ln n}{n} \stackrel{L}{=} \lim_{n\to\infty} \frac{1}{n} = 0$$

sequence is bounded

$$0 < a_n < \frac{\ln 5}{5}$$

7. Which of the following integrals gives the surface area obtained by rotating the curve $y = \sin\left(\frac{x}{2}\right)$, $0 \le x \le \frac{\pi}{2}$ about the y axis?

(a) $\int_0^{\sqrt{2}/2} 4\pi \arcsin y \sqrt{1 + \frac{4}{1-y^2}}\, dy$

(b) $\int_0^{\sqrt{2}/2} 2\pi y \sqrt{1 + \frac{4}{1-y^2}}\, dy$

(c) $\int_0^{\sqrt{2}/2} 2\pi \arcsin y \sqrt{1 + \frac{4}{1-y^2}}\, dy$

(d) $\int_0^{1} 4\pi \arcsin y \sqrt{1 + \frac{4}{1-y^2}}\, dy$

(e) $\int_0^{1} 2\pi y \sqrt{1 + \frac{4}{1-y^2}}\, dy$

$y = \sin\left(\frac{x}{2}\right)$

$\arcsin y = \frac{x}{2}$

$x = 2\arcsin y$

$\frac{dx}{dy} = \frac{2}{\sqrt{1-y^2}}$ $\quad 0 \le x \le \frac{\pi}{2}$

$0 \le y \le \frac{\sqrt{2}}{2}$

$A = \int_0^{\frac{\sqrt{2}}{2}} 2\pi \cdot 2\arcsin y \sqrt{1 + \frac{4}{1-y^2}}\, dy$

8. Find s_4, the fourth partial sum, of the series $\sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{3}\right)$.

(a) $s_4 = \frac{1}{2}$

(b) $s_4 = -\frac{3}{2}$

(c) $s_4 = -\frac{\sqrt{3}}{2}$

(d) $s_4 = -\frac{1}{2}$

(e) $s_4 = -1 - \frac{\sqrt{3}}{2}$

$S_4 = \cos\frac{\pi}{3} + \cos\frac{2\pi}{3} + \cos\pi + \cos\frac{4\pi}{3}$

$= \frac{1}{2} - \frac{1}{2} - 1 - \frac{1}{2}$

$= -\frac{3}{2}$

9. Compute $\int_2^3 \frac{x^3}{x-1}\, dx$.

(a) $\frac{29}{6} - \ln 2$

(b) $\frac{29}{6} + \ln 2$

(c) $\frac{59}{6} - \ln 2$

(d) $\frac{59}{6} + \ln 2$

(e) $\frac{5}{2} + \ln 2$

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
x-1 & x^3 \\
 & x^3 - x^2 \\
 & x^2 \\
 & x^2 - x \\
 & x \\
 & x - 1 \\
 & 1
\end{array}
$$

$\int_2^3 \left(x^2 + x + 1 + \frac{1}{x-1}\right) dx = \left(\frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1|\right)\Big|$

$= \ln 2 + \frac{59}{6}$

10. The sequence $a_n = 2\ln(7n+3) - \ln(5n^2+1)$

(a) diverges

(b) converges to $\ln\left(\frac{5}{49}\right)$

(c) converges to $\ln\left(\frac{7}{5}\right)$

(d) converges to $\ln\left(\frac{7}{25}\right)$

(e) converges to $\ln\left(\frac{49}{5}\right)$

$$\lim_{n\to\infty}\left(\ln(7n+3)^2 - \ln(5n^2+1)\right)$$

$$\lim_{n\to\infty}\ln\left(\frac{(7n+3)^2}{5n^2+1}\right) = \ln\frac{49}{5}$$

11. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \dfrac{2n+3}{n+5}$, find a_3 as well as the sum, S, of the series $\sum_{n=1}^{\infty} a_n$.

(a) $a_3 = \frac{9}{8}$ and $S = 1$.

(b) $a_3 = \frac{71}{24}$ and $S = 2$.

(c) $a_3 = \frac{1}{8}$ and $S = 2$.

(d) $a_3 = \frac{1}{8}$ and the series diverges.

(e) $a_3 = \frac{9}{8}$ and the series diverges.

$$a_3 = S_3 - S_2$$

$$= \frac{9}{8} - \frac{7}{7}$$

$$a_3 = \frac{1}{8}$$

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty}\frac{2n+3}{n+5} = 2$$

12. Which of the following integrals results after performing an appropriate trigonometric substitution for $\int_0^{1/2} x^2\sqrt{1+4x^2}\,dx$?

(a) $\frac{1}{8}\int_0^{\pi/4} \tan^2\theta\sec^3\theta\,d\theta$

(b) $\frac{1}{8}\int_0^{\pi/4} \tan^2\theta\sec\theta\,d\theta$

(c) $4\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$

(d) $8\int_0^{\pi/4} \tan^2\theta\sec^3\theta\,d\theta$

(e) $\frac{1}{4}\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$

$2x = \tan\theta$ $\quad x = \frac{1}{2},\ \theta = \frac{\pi}{4}$; $\quad x = 0,\ \theta = 0$

$dx = \frac{1}{2}\sec^2\theta\,d\theta$

$$\int_0^{\frac{\pi}{4}} \frac{1}{4}\tan^2\theta\sqrt{1+\tan^2\theta}\,\frac{1}{2}\sec^2\theta\,d\theta$$

$$\frac{1}{8}\int_0^{\frac{\pi}{4}} \tan^2\theta\sec^3\theta\,d\theta$$

13. The integral $\int_{-1}^{2} \frac{dx}{x^3}$ $= \int_{-1}^{0} \frac{dx}{x^3} + \int_{0}^{2} \frac{dx}{x^3}$

(a) converges to $\frac{3}{4}$

(b) diverges

(c) converges to $\frac{3}{2}$

(d) converges to $\frac{3}{8}$

(e) converges to $\frac{7}{32}$

diverges since $\int_0^2 \frac{dx}{x^3} = \lim_{t\to 0^-} \int_t^2 \frac{dx}{x^3}$

$= \lim_{t\to 0^-} \left. \frac{-1}{2x^2} \right|_t^2$

$= \lim_{t\to 0^-} \left(\frac{-1}{8} + \frac{1}{2x^2} \right)$

$= \infty$

14. After making an appropriate trigonometric substitution, which of the following integrals is equivalent to $\int \sqrt{-x^2 + 2x + 3}\, dx$?

(a) $2 \int \sec\theta \, d\theta$

(b) $4 \int \sec^3\theta \, d\theta$

(c) $4 \int \sec\theta \tan^2\theta \, d\theta$

(d) $2 \int \cos\theta \, d\theta$

(e) $4 \int \cos^2\theta \, d\theta$

$-x^2 + 2x + 3 = -(x^2 + 2x + 1) + 4$

$= -(x+1)^2 + 4$

$\int \sqrt{4 - (x-1)^2}\, dx$

let $x - 1 = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$\int \sqrt{4 - 4\sin^2\theta}\; 2\cos\theta \, d\theta$

$= 4 \int \cos^2\theta \, d\theta$

15. Find the length of the curve $y = \sqrt[3]{x^2}$, $0 \le y \le 1$.

(a) $\frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$

(b) $\frac{2}{3}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$

(c) $\frac{3}{2}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$

(d) $\frac{2}{3}\left(\left(\frac{5}{3}\right)^{3/2} - 1\right)$

(e) $\frac{4}{9}\left(\left(\frac{5}{3}\right)^{3/2} - 1\right)$

$y = x^{\frac{2}{3}}$

$x = y^{\frac{3}{2}}$

$\frac{dx}{dy} = \frac{3}{2}\sqrt{y}$

$\int_0^1 \sqrt{1 + \frac{9}{4}y}\, dy$

$= \frac{4}{9} \cdot \frac{2}{3} \left. \left(1 + \frac{9}{4}y\right)^{\frac{3}{2}} \right|_0^1$

$= \frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$

PART II: Work Out

16. Consider the sequence $a_n = \dfrac{7n}{8n+4}$.

a.) (2 pts) Find the limit of a_n.

$$\lim_{n\to\infty} \frac{7n}{8n+4} \stackrel{L}{=} \lim_{x\to\infty} \frac{7}{8} = \frac{7}{8}$$

b.) (3 pts) Find the sum of the series $\sum_{n=1}^{\infty} a_n$ or explain why it diverges.

$\sum_{n=1}^{\infty} a_n$ diverges by Test for divergence since $\lim_{n\to\infty} a_n = \frac{7}{8} \neq 0$

17. (8 pts) Compute $\int_e^{\infty} \frac{\ln x}{x^2}\, dx$ or show that it diverges.

Parts: $u = \ln x \quad dv = \dfrac{dx}{x^2}$

$du = \dfrac{dx}{x}, \quad v = -\dfrac{1}{x}$

$$\int \frac{\ln x}{x^2}\, dx = uv - \int v\, du$$

$$= -\frac{\ln x}{x} + \int \frac{dx}{x^2}$$

$$= -\frac{\ln x}{x} - \frac{1}{x}$$

$$\int_e^{\infty} \frac{\ln x}{x^2}\, dx = \lim_{t\to\infty} \left(-\frac{\ln x}{x} - \frac{1}{x}\right)\Big|_e^t$$

$$= \lim_{t\to\infty} \left(-\frac{\ln t}{t} - \frac{1}{t} - \left(-\frac{\ln e}{e} - \frac{1}{e}\right)\right)$$

$$\lim_{t\to\infty} \frac{\ln t}{t} \stackrel{L}{=} \lim_{t\to\infty} \frac{\frac{1}{t}}{1} = 0$$

$$= \frac{2}{e}$$

18. a.) (4 pts) Find the partial fraction decomposition for $\frac{-2}{(2n+1)(2n-1)}$.

$$\frac{-2}{(2n+1)(2n-1)} = \frac{A}{2n+1} + \frac{B}{2n-1}$$

$$\boxed{\frac{-2}{(2n+1)(2n-1)} = \frac{1}{2n+1} - \frac{1}{2n-1}}$$

$$-2 = A(2n-1) + B(2n+1)$$

$$n = -\frac{1}{2} ; \quad -2 = A(-2) \quad A = 1$$

$$n = \frac{1}{2} : \quad -2 = B(2) \quad B = -1$$

b.) (4 pts) Find a formula for s_n, the nth partial sum of the series $\sum_{n=1}^{\infty} \frac{-2}{(2n+1)(2n-1)}$.

$$\sum_{n=1}^{\infty} \left(\frac{1}{2n+1} - \frac{1}{2n-1} \right)$$

$$S_n = \underbrace{\cancel{\frac{1}{3}} - 1}_{a_1} + \underbrace{\cancel{\frac{1}{5}} - \cancel{\frac{1}{3}}}_{a_2} + \underbrace{\cancel{\frac{1}{7}} - \cancel{\frac{1}{5}}}_{a_3} + \underbrace{\cancel{\frac{1}{9}} - \cancel{\frac{1}{7}}}_{a_4} + \dots + \underbrace{\cancel{\frac{1}{2n-1}} - \cancel{\frac{1}{2n-3}}}_{a_{n-1}} + \underbrace{\frac{1}{2n+1} - \cancel{\frac{1}{2n-1}}}_{a_n}$$

$$\boxed{S_n = -1 + \frac{1}{2n+1}}$$

c.) (4 pts) Find $\sum_{n=1}^{\infty} \frac{-2}{(2n+1)(2n-1)}$.

$$\sum_{n=1}^{\infty} \frac{-2}{(2n+1)(2n-1)} = \lim_{n \to \infty} \left(-1 + \frac{1}{2n+1} \right)$$

$$= \boxed{-1}$$

19. Consider the surface obtained by rotating the curve $y = \ln(2x+3)$, $1 \le x \le 3$, about the x-axis.

a.) (5 pts) Set up but do not evaluate an integral in terms of x that gives the area of the surface.

$$\frac{dy}{dx} = \frac{2}{2x+3}$$

$$SA = \int_1^3 2\pi \ln(2x+3)\sqrt{1+\frac{4}{(2x+3)^2}}\,dx$$

b.) (5 pts) Set up but do not evaluate an integral in terms of y that gives the area of the surface.

$$y = \ln(2x+3) \qquad 1 \le x \le 3 \qquad \ln 5 \le y \le \ln 9$$

$$e^y = 2x+3$$

$$x = \frac{e^y - 3}{2}$$

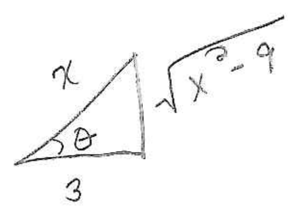
$$SA = \int_{\ln 5}^{\ln 9} 2\pi y\sqrt{1+\left(\frac{e^y}{2}\right)^2}\,dy$$

20. (9 pts) Find $\int \frac{1}{x^4\sqrt{x^2-9}}\,dx$. Express your answer without the use of trig or inverse trig functions.

$$x = 3\sec\theta$$

$$dx = 3\sec\theta\tan\theta\,d\theta$$

$$\int \frac{3\sec\theta\tan\theta\,d\theta}{81\sec^4\theta\sqrt{9\sec^2\theta-9}}$$



$$\frac{1}{27}\int \frac{\tan\theta\,d\theta}{\sec^3\theta\cdot 3\tan\theta}$$

$$\frac{1}{81}\int \frac{d\theta}{\sec^3\theta} = \frac{1}{81}\int \cos^3\theta\,d\theta$$

$$= \frac{1}{81}\int (1-\sin^2\theta)\cos\theta\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{1}{81}\int (1-u^2)\,du$$

$$= \frac{1}{81}\left(u - \frac{u^3}{3}\right) + C$$

$$\rightarrow \frac{1}{81}\left(\sin\theta - \frac{1}{3}\sin^3\theta\right)dt$$

$$\boxed{\frac{1}{81}\left(\frac{\sqrt{x^2-9}}{x} - \frac{1}{3}\left(\frac{\sqrt{x^2-9}}{x}\right)^3\right) + C}$$

21. (11 pts) Find $\int \frac{3x^2+x-24}{(x-1)(x^2+4)}\,dx$

$$\frac{3x^2+x-24}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

$$3x^2+x-24 = A(x^2+4)+(Bx+C)(x-1)$$

$$x=1: \quad -20 = A(5) \qquad A=-4$$

$$3x^2+x-24 = -4x^2-16+Bx^2-Bx+Cx-C$$

$$= (-4+B)x^2+(C-B)x+-16-C$$

$$3=-4+B$$

$$B=7$$

$$-24=-16-C$$

$$C=8$$

$$\int\left(\frac{-4}{x-1}+\frac{7x+8}{x^2+4}\right)dx = \int\left(\frac{-4}{x-1}+\frac{7x}{x^2+4}+\frac{8}{x^2+4}\right)dx$$

$$-4\ln|x-1|+\frac{7}{2}\ln(x^2+4)+\frac{8}{2}\arctan\frac{x}{2}+C$$