

**MATH 152 Spring 2016  
COMMON EXAM III - VERSION A**

LAST NAME: \_\_\_\_\_ FIRST NAME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

UIN: \_\_\_\_\_

**DIRECTIONS:**

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form*.

THE AGGIE CODE OF HONOR

**“An Aggie does not lie, cheat or steal, or tolerate those who do.”**

Signature: \_\_\_\_\_

**DO NOT WRITE BELOW!**

Question	Points Awarded	Points
1-15		45
16		12
17		12
18		8
19		9
20		14
Total		100

**PART I: Multiple Choice. 3 points each.**

1. The region in  $\mathbb{R}^3$  described by the equation  $y = 8$  is:

- (a) A line parallel to the  $x$ -axis.
- (b) A plane parallel to the  $yz$  plane passing through the point  $(0, 8, 0)$ .
- (c) The point  $(0, 8, 0)$ .
- (d) A plane parallel to the  $xz$  plane passing through the point  $(0, 8, 0)$ .
- (e) A line parallel to the  $z$ -axis.

2. Find the radius of convergence,  $R$ , and the interval of convergence,  $I$ , for  $\sum_{n=1}^{\infty} \frac{(x+5)^n n!}{3^n}$ .

- (a)  $R = 3, I = (-8, -2)$
- (b)  $R = 0, I = \{-5\}$
- (c)  $R = \infty, I = \{-5\}$
- (d)  $R = 3, I = (2, 8)$
- (e)  $R = \infty, I = (-\infty, \infty)$

3. Find the third degree Taylor Polynomial,  $T_3(x)$ , for  $f(x) = \cos(2x)$  at  $a = \frac{\pi}{6}$ .

- (a)  $T_3(x) = \frac{1}{2} - \sqrt{3} \left(x - \frac{\pi}{6}\right) - \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3} \left(x - \frac{\pi}{6}\right)^3$
- (b)  $T_3(x) = \frac{1}{2} - \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right) - \frac{1}{4} \left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{12} \left(x - \frac{\pi}{6}\right)^3$
- (c)  $T_3(x) = \frac{1}{2} - \sqrt{3} \left(x - \frac{\pi}{6}\right) - 2 \left(x - \frac{\pi}{6}\right)^2 + 4\sqrt{3} \left(x - \frac{\pi}{6}\right)^3$
- (d)  $T_3(x) = \frac{\sqrt{3}}{2} - \left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4} \left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{3} \left(x - \frac{\pi}{6}\right)^3$
- (e)  $T_3(x) = \frac{1}{2} - \sqrt{3} \left(x - \frac{\pi}{6}\right) + \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3} \left(x - \frac{\pi}{6}\right)^3$

4. Which of the following is a Maclaurin series for  $f(x) = x^3 e^{-x^2}$ ?

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n)!}$

(b)  $\sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$

(c)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{n!}$

(d)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{5n}}{n!}$

(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!}$

5. Find the equation of the sphere whose diameter has endpoints  $(2, 1, 6)$  and  $(4, -3, 8)$ .

(a)  $(x + 3)^2 + (y - 1)^2 + (z + 7)^2 = 6$

(b)  $(x - 3)^2 + (y + 1)^2 + (z - 7)^2 = 24$

(c)  $(x - 3)^2 + (y + 1)^2 + (z - 7)^2 = 6$

(d)  $(x - 3)^2 + (y - 1)^2 + (z - 7)^2 = 24$

(e)  $(x - 6)^2 + (y + 2)^2 + (z - 14)^2 = 24$

6. Write  $f(x) = \frac{x^3}{8+x}$  as a power series centered at 0.

(a)  $\sum_{n=0}^{\infty} \frac{x^{n+3}}{8^{n+1}}$

(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{64^n}$

(c)  $\sum_{n=0}^{\infty} \left(-\frac{x}{8}\right)^{n+3}$

(d)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{8^{n+1}}$

(e)  $\sum_{n=0}^{\infty} \left(-\frac{x}{8}\right)^{3n}$

7. Using the Alternating Series Estimation Theorem, what is the smallest number of terms we must use to approximate

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \text{ with error less than } \frac{1}{120}?$$

- (a)  $n = 11$
- (b)  $n = 9$
- (c)  $n = 10$
- (d)  $n = 12$
- (e)  $n = 13$

8. Which of the following statements is true of the series  $\sum_{n=1}^{\infty} \frac{n^3}{n^4 + n^2 + 1}$ ?

I. Diverges by comparison test with  $\sum_{n=1}^{\infty} \frac{1}{n}$

II. Diverges by limit comparison test with  $\sum_{n=1}^{\infty} \frac{1}{n}$

III. Diverges by Test for Divergence

- (a) Only I and II are true
- (b) Only I is true
- (c) Only III is true
- (d) Only II is true
- (e) None of I, II and III is true.

9. Find  $f^{(23)}(3)$ , that is, the 23<sup>rd</sup> derivative of  $f(x)$  at  $x = 3$ , if  $f(x) = \sum_{n=0}^{\infty} \frac{(-3)^{n+2}}{(2n)!} (x - 3)^n$ .

- (a)  $\frac{(-3)^{24}(23)!}{(46)!}$
- (b) 0
- (c)  $\frac{(-3)^{25}}{(46)!}$
- (d)  $\frac{(-3)^{25}(24)!}{(46)!}$
- (e)  $\frac{(-3)^{25}(23)!}{(46)!}$

10. 
$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{2\pi}{3}\right)^{2n+1}}{(2n+1)!} =$$

(a)  $\frac{\sqrt{3}}{2}$

(b)  $-\frac{1}{2}$

(c)  $-\frac{\sqrt{3}}{2}$

(d)  $\frac{1}{2}$

(e)  $e^{2\pi/3}$

11. For which of the following series is the Ratio Test inconclusive?

I)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 3}$

II)  $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

III)  $\sum_{n=1}^{\infty} n4^n$

(a) I only

(b) II only

(c) I and II only

(d) II and III only

(e) I, II, and III

12. Suppose it is known that  $\sum_{n=0}^{\infty} c_n x^n$  converges when  $x = -3$  and diverges when  $x = 5$ . Which of the following is certain to be true?

(a)  $\sum_{n=0}^{\infty} (-1)^n c_n 7^n$  is divergent

(b)  $\sum_{n=0}^{\infty} c_n (-5)^n$  is convergent

(c)  $\sum_{n=0}^{\infty} (-1)^n c_n 4^n$  is divergent

(d)  $\sum_{n=0}^{\infty} c_n 4^n$  is divergent

(e)  $\sum_{n=0}^{\infty} c_n 3^n$  is convergent

13.  $\int \cos(2x^2) dx =$

- (a)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+1}}{(4n+1)(2n)!}$
- (b)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+4}}{(4n+4)(2n)!}$
- (c)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+2}}{(2n+1)(2n)!}$
- (d)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+2}}{(2n+1)(2n)!}$
- (e)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+3}}{(4n+3)(2n+1)!}$

14.  $\sum_{n=0}^{\infty} 5(-6)^n x^{2n} =$

- (a)  $\frac{-5}{1+6x^2}$ , where  $|x| < \sqrt{6}$
- (b)  $\frac{5}{1+6x^2}$ , where  $|x| < \frac{1}{\sqrt{6}}$
- (c)  $\frac{5}{(1-6x)^2}$ , where  $|x| < \sqrt{6}$
- (d)  $\frac{-5}{1+6x^2}$ , where  $|x| < \frac{1}{\sqrt{6}}$
- (e)  $\frac{5}{(1+6x)^2}$ , where  $|x| < \sqrt{6}$

15.  $\sum_{n=1}^{\infty} \frac{\arctan(n) + 4}{n^8}$  is:

- (a) Convergent by the Comparison Test with  $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2} + 4}{n^8}$
- (b) Convergent by the Comparison Test with  $\sum_{n=1}^{\infty} \frac{1}{n^8}$
- (c) Divergent by the Limit Comparison Test with  $\sum_{n=1}^{\infty} \frac{1}{n^8}$
- (d) Divergent by the Test for Divergence
- (e) Divergent by the Integral Test

**PART II: Work Out**

16. For the power series  $\sum_{n=2}^{\infty} \frac{(x+1)^n}{(-3)^n \ln(n)}$ :

a.) (4 pts) Find the radius of convergence.

b.) (8 pts) Find the interval of convergence. You must test the endpoints for convergence.

17. For  $f(x) = \frac{2}{x^2}$ :

a.) (9 pts) Find the Taylor series at  $a = 5$ .

b.) (3 pts) Find the radius of convergence for the Taylor series found in part a.).

18. (8 pts) Consider  $\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^3 - 1}$ . Determine whether the series converges absolutely, converges but not absolutely, or diverges. Fully support your conclusion.

19. Consider  $\sum_{n=1}^{\infty} \frac{1}{(n+1)(\ln(n+1))^2}$ .

a.) (5 pts) Prove the series converges.

b.) (4 pts) If we used  $s_3$ , the third partial sum, to approximate the sum  $s$  of the series, use the Remainder Estimate for the Integral Test to find an upper bound on the remainder,  $R_3 = s - s_3$ .

20. (i) (9 pts) Find a power series about zero for  $f(x) = \ln(4 + x^2)$ .

(ii) (3 pts) What is the radius of convergence of the series above?

(iii) (2 pts) Using (i), find a power series about zero for  $g(x) = x^3 \ln(4 + x^2)$ .