Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Find the area of the region bounded by the curves

$$x = 0, \quad x = 1 + y^2, \quad y = 1, \quad y = 3.$$

(A) 11

(B) $\frac{40}{3}$

(C) 12

(D) $\frac{32}{3}$

(E) 10

2. Find the average value of the function $f(x) = e^{-3x}$ on the interval $[0, 2]$.

(A) $\frac{1}{6}(1 - e^{-6})$

(B) $\frac{3}{2}(1 + e^{6})$

(C) 0

(D) $\frac{1}{3}(1 - e^{-6})$

(E) $\frac{1}{2}(e^{-6} - 1)$

3. Calculate $\int_0^{\pi/4} \sin^2 x \, dx$.

(A) $\frac{\pi}{8} - \frac{1}{2}$

(B) $\frac{\pi}{4} - \frac{1}{2}$

(C) $\frac{\pi}{8} - \frac{1}{4}$

(D) $\frac{\pi}{8} + \frac{1}{2}$

(E) $\frac{\pi}{4} + \frac{1}{4}$

4. An object is moved along the x-axis by a force of magnitude $F(x) = \frac{1}{1+x^2}$. How much work is done as the object moves from $x = 0$ to $x = 1$?

(A) π

(B) $\frac{\pi}{16}$

(C) $\ln 2$

(D) $\frac{\pi}{4}$

(E) $\ln 8 - \ln 2$

5. The area bounded by the curves $x^2 = y$ and $x + y = 2$ is

(A) 5

(B) $\frac{3}{2}$

(C) $\frac{9}{2}$

(D) π

(E) $\frac{\pi}{2}$

6. A trigonometric substitution converts the integral $\int \frac{x}{(3 - 2x - x^2)^{1/2}} dx$ to

(A) $\int (3\cos\theta + 2)\, d\theta$

(B) $\int (2\sin\theta - 1)\, d\theta$

(C) $\int (2\sin^2\theta - \cos\theta)\, d\theta$

(D) $\int (2\tan\theta - 1)\, d\theta$

(E) $\int 2\tan^{-1}\theta\, d\theta$

7. Suppose that $f(0) = 3$ and $f(2) = 4$ and $\int_0^2 x^2 f(x)\,dx = 5$. What is $\int_0^2 x^3 f'(x)\,dx$? (*Hint:* Use integration by parts. Assume that $f(x)$ is a differentiable function and that $f'(x)$ is continuous.)

(A) 60

(B) 47

(C) 33

(D) 27

(E) 17

8. The region bounded by the curves $x = 0, \quad x = 1+y, \quad y = 0, \quad y = 2$ is rotated about the y-axis. Find the volume of the resulting solid.

(A) $\frac{26\pi}{3}$

(B) $\frac{80\pi}{3}$

(C) $\frac{22\pi}{3}$

(D) $\frac{32\pi}{3}$

(E) $\frac{62\pi}{3}$

9. Calculate $\int_1^e \frac{\ln x}{x^2}\,dx$.

(A) $e^2 - 1$

(B) 0

(C) 1

(D) $1 - \frac{2}{e}$

(E) $2 - e^2$

10. The base of a solid is the triangle with vertices $(0,0)$, $(1,1)$, and $(1,-1)$. The cross sections perpendicular to the x-axis are squares. Find the volume.

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{4}{3}$

(D) $\frac{16}{3}$

(E) $\frac{32}{3}$

Part II: Write Out (10 points each)

Show all your work. Appropriate partial credit will be given. You may not use a calculator.

11. Evaluate $\int x^2 \sin(4x)\, dx$.

12. Evaluate $\int \frac{\sin^3 x}{\cos^4 x}\, dx$.

13. Set up (but DO NOT EVALUATE) the integrals to compute the volumes of the indicated solids of revolution. CLEARLY INDICATE IN EACH CASE WHETHER YOU ARE WRITING A CYLINDER-SHELL FORMULA OR A DISKS/WASHERS FORMULA.

    (a) Revolve the region bounded by $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$ about the line $x = 0$ (the y-axis).

    (b) Revolve the region bounded by $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$ about the line $y = 2$.

14. Evaluate $\displaystyle\int \frac{1}{\sqrt{x^2 - 9}}\, dx$.

15. Do **ONE** of the following [(A) or (B)]. CIRCLE THE LETTER of the one you want graded!

(A) A 10-kilogram object at ground level is attached by a cable with a mass density of $\frac{1}{4}$ kg/m to a winch at the top of a 40-meter high building. How much work (in joules) is required to crank this load up to the roof? (The acceleration of gravity in MKS units is $g = 9.8$.)

(B) A tank (trough) 8 feet long has cross sections that are isosceles triangles (with base side on top) whose base and altitude are both 3 feet. If the tank is initially full of water, how much work is required to pump all the water out over the top? (Water weighs 62.5 pounds per cubic foot.)