Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. A cube, with edges of length 2 meters, sits on the flat bottom of a pool of water that is 5 meters deep. Find the force due to water pressure on any one vertical side of the cube. (Let ρ be the mass density of water and g be the acceleration of gravity; $\rho = 1000$ kg/m^3 and $g = 9.8$ m/s^2, but you don't need to use that.)

   (A) $20\rho g$

   (B) $50\rho g$

   (C) $4\rho g$

   (D) $12\rho g$

   (E) $16\rho g$

2. Find the force due to water pressure on the top surface of the cube in the previous problem.

   (A) $20\rho g$

   (B) $50\rho g$

   (C) $4\rho g$

   (D) $12\rho g$

   (E) $16\rho g$

3. The improper integral $\int_0^4 \frac{dx}{\sqrt{x}}$

(A) converges to the value 4

(B) diverges, by comparison with the integral $\int_0^4 \frac{dx}{x^2}$

(C) converges to the value 3

(D) converges to the value $\frac{1}{2}$

(E) converges, by comparison with the integral $\int_0^4 \frac{dx}{x^2}$

4. Set up the integral for the arc length of the curve $y = \dfrac{x^3}{6}$ between $x = 1$ and $x = 2$.

(A) $\dfrac{\pi}{36} \displaystyle\int_1^2 x^6 \, dx$

(B) $\displaystyle\int_1^2 \sqrt{1 + \frac{x^4}{4}} \, dx$

(C) $\displaystyle\int_1^2 \sqrt{1 + \frac{x^2}{2}} \, dx$

(D) $\dfrac{\pi}{3} \displaystyle\int_1^2 x^4 \, dx$

(B) $\displaystyle\int_1^2 \sqrt{1 + \frac{x^6}{36}} \, dx$

5. The improper integral $\displaystyle\int_e^\infty \frac{dx}{x \ln x}$ [**Hint:** Let $u = \ln x$.]

(A) converges to the value -1

(B) converges to the value 0

(C) converges, by comparison with the integral $\displaystyle\int_e^\infty \frac{dx}{x}$

(D) converges to the value $-e$

(E) diverges to $+\infty$

6. Set up the integral for the arc length of the parametrized curve segment

$$x(t) = \sin 2t, \quad y(t) = 3\cos 2t, \qquad 0 \le t \le \frac{\pi}{4}.$$

(A) $\displaystyle\int_0^{\pi/4} \left(1 + \sqrt{4\cos^2 2t + 36\sin^2 2t}\right) dt$

(B) $\displaystyle\int_0^{\pi/4} \sqrt{1 + 2\cos 2t + 9\sin 2t}\, dt$

(C) $\displaystyle\int_0^{\pi/4} \sqrt{4\cos^2 2t + 36\sin^2 2t}\, dt$

(D) $\displaystyle\int_0^{\pi/4} \sqrt{1 + \sin^2 2t + 9\cos^2 2t}\, dt$

(E) $\displaystyle\int_0^{\pi/4} \sqrt{4\sin^2 2t + 6\cos^2 2t}\, dt$

7. To find the partial-fraction decomposition of $\dfrac{3x^4+9x^3+9x^2+x+8}{(x^2+4x+5)(x-1)^2(x+2)}$ you would start from the form

(A) $\dfrac{A}{x+2}+\dfrac{B}{(x-1)^2}+\dfrac{Cx+D}{x^2+4x+5}+\dfrac{Ex+F}{(x^2+4x+5)^2}$

(B) $\dfrac{A}{x+2}+\dfrac{B}{x-1}+\dfrac{Cx+D}{x^2+4x+5}$

(C) $\dfrac{A}{x+2}+\dfrac{B}{(x-1)^2}+\dfrac{C}{(x-1)}+\dfrac{Dx+E}{x^2+4x+5}$

(D) $\dfrac{A}{x+2}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x^2+4x+5}$

(E) $3x^2+\dfrac{A}{x+2}+\dfrac{B}{(x-1)^2}+\dfrac{Cx+D}{x^2+4x+5}$

8. An integrating factor for the differential equation $\dfrac{dy}{dx}+\dfrac{3y}{x}=\cos x$ is

(A) $e^{\sin x}$

(B) x^3

(C) $3x$

(D) $3\ln x$

(E) $e^{3/x}$

9. The improper integral $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^{3/2}}\,dx$

(A) converges, by comparison with the integral $\displaystyle\int_1^\infty \frac{1}{x^{3/2}}\,dx$

(B) diverges, by comparison with the integral $\displaystyle\int_1^\infty \frac{1}{x^{3/2}}\,dx$

(C) diverges, by comparison with the integral $\displaystyle\int_1^\infty \sin^2 x\,dx$

(D) converges to the value 0

(E) converges to the value $-\pi$

10. Suppose that $y(t)$ is the solution to the initial value problem $\dfrac{dy}{dt} = y^2, \quad y(0) = \dfrac{1}{2}$. What is $y(5)$?

(A) $-\dfrac{1}{3}$

(B) $-\dfrac{1}{5}$

(C) $\dfrac{1}{7}$

(D) $\dfrac{1}{2}e^5$

(E) $\dfrac{1}{2}e^{5/2}$

Part II: Write Out (10 points each)

Show all your work. Appropriate partial credit will be given. You may not use a calculator.

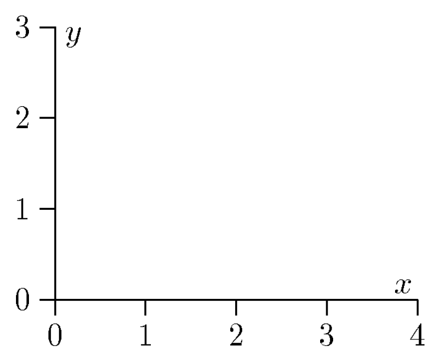
11. The curve $y = \sqrt{x}$, between $x = 0$ and $x = 4$, is rotated about the x-axis. Find the surface area of the resulting surface.

12. A tank has volume 1000 liters and initially contains pure fresh water. Water containing 10 grams of salt per liter runs into the tank at a rate of 20 L/min. The salt water mixes instantly and completely with the fresh water, and the mixture drains out the bottom at a rate of 20 L/min. Find a formula for $S(t)$, the number of grams of salt in the tank after t minutes.

13. In this problem we shall use the trapezoid rule with $n = 4$ to approximate $\int_1^3 \sqrt{x}\,dx$. The three parts of the problem can be done in any order. Note: At no time should you be using the antiderivative of the function $\sqrt{x}$.

(a) Write out the trapezoid-rule approximation T_4 to $\displaystyle\int_1^3 \sqrt{x}\,dx$.
DO NOT SIMPLIFY; the purpose of this question is to show that you understand the formula, not that you can do arithmetic.

(b) Sketch the graph of $f(x) = \sqrt{x}$, $1 \le x \le 3$, along with the approximating trapezoids.



(c) Using the error-bound formula

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \quad \text{where } |f''(x)| \le K \text{ for } a \le x \le b,$$

find an upper bound for the error in using T_4 to approximate $\displaystyle\int_1^3 \sqrt{x}\,dx$.

14. Evaluate $\int \frac{3x^2 - x + 8}{x^3 + 4x}\, dx$.

15. A plate of uniform density occupies the region bounded by $y = 1 + x^3$, $y = 0$, $x = 0$, and $x = 2$. Find $\overline{x}$, the x component of the centroid (center of mass) of the plate.