Part I: Multiple Choice (4 points each)

There is no partial credit. You may not use a calculator.

1. Suppose that the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is given by $s_n = 1 - \dfrac{\ln n}{n}$. Then the series $\sum_{n=1}^{\infty} a_n$

(A) converges to 0 .

(B) converges to 1 .

(C) converges to -1 .

(D) diverges to $+\infty$.

(E) diverges to $-\infty$.

2. If $\vec{a} = \langle 1, 1, 1 \rangle$ and $\vec{b} = \langle 1, -2, 3 \rangle$, then $3\vec{a} + \vec{b}$ is

(A) $\langle 0, 3, -2 \rangle$

(B) $\langle 2, -1, 4 \rangle$

(C) $\langle 2, 1, 3 \rangle$

(D) $\langle 4, 2, 0 \rangle$

(E) $\langle 4, 1, 6 \rangle$

3. $\lim_{n\to\infty} \dfrac{\cos(n^{97})}{\sqrt{n}}$ equals

(A) 0

(B) ± 1

(C) 97

(D) $\frac{97}{2}$

(E) It does not exist.

4. Compute the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{5^n}$, if it converges.

(A) $\dfrac{5}{3}$

(B) $\dfrac{2}{5}$

(C) $\dfrac{5}{2}$

(D) $\dfrac{3}{5}$

(E) It diverges.

5. The region of $\mathbf{R}^3$ represented by the equation $xyz = 0$ consists of

(A) the x-axis, the y-axis, and the z-axis.

(B) the x-axis and the yz-plane.

(C) the xy-plane, the yz-plane, and the xz-plane.

(D) a line.

(E) none of these.

6. The Maclaurin series (Taylor series around 0) of $f(x) = x^2 e^{-3x}$ is

(A) $\displaystyle\sum_{n=0}^{\infty} \frac{-x^{n+2}}{3^n n!}$

(B) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{3^n n!}$

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n x^{n+2}}{n!}$

(D) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{(3n)!}$

(E) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+2}}{(3n)!}$

7. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n + e^{-2n}}{n^2 - e^{-n}}$ is

(A) convergent, by the ratio test.

(B) divergent, by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) convergent, by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

(D) divergent, by comparison with $\displaystyle\sum_{n=1}^{\infty} e^n$.

(E) convergent, by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n}$.

8. The sequence $\left\{\frac{n^2-1}{n}\right\}_{n=1}^{\infty}$ is

(A) increasing and bounded above.

(B) decreasing and bounded above.

(C) increasing and bounded below.

(D) decreasing and bounded below.

(E) none of these

9. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is

(A) absolutely convergent.

(B) divergent, because the exponent $\frac{1}{2}$ is less than 1.

(C) convergent, by the ratio test.

(D) convergent, by the alternating series test.

(E) divergent, by the alternating series test.

10. What is the Maclaurin series of $f(x) = x^2$?

(A) $x^2 + x^3 + x^4 + \cdots$

(B) x^2

(C) $\sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$

(D) $\sum_{n=0}^{\infty} (x-1)^2$

(E) It does not exist.

11. Use the Maclaurin series (Taylor series around 0) for e^{-x^3} to compute

$$\lim_{x\to 0} \frac{e^{-x^3} - 1 + x^3}{x^6}.$$

(A) $-\frac{3}{6!}$

(B) $\frac{3}{6!}$

(C) $\frac{1}{6!}$

(D) $-\frac{1}{2}$

(E) $\frac{1}{2}$

12. The Taylor series of $f(x) = \frac{1}{x}$ around (i.e., centered at) the point $x = -3$ is

(A) $-\sum_{n=0}^{\infty} \frac{(x+3)^n}{3^{n+1}}$

(B) $\sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{3^n}$

(D) $\sum_{n=0}^{\infty} (x+3)^n$

(E) $\sum_{n=0}^{\infty} (-3)^n x^n$

13. Use the Maclaurin series for $\cos(t^2)$ to find the Maclaurin series for $\int_0^x \cos(t^2)\,dt$.

(A) $-\dfrac{4x^3}{2!} + \dfrac{8x^7}{4!} - \cdots$

(B) $\dfrac{x^2}{2 \cdot 0!} - \dfrac{x^6}{6 \cdot 4!} + \dfrac{x^{10}}{10 \cdot 8!} - \cdots$

(C) $x - \dfrac{x^3}{3!} + \dfrac{x^7}{7!} - \cdots$

(D) $x - \dfrac{x^5}{5 \cdot 2!} + \dfrac{x^9}{9 \cdot 4!} - \cdots$

(E) $x - \dfrac{x^3}{3 \cdot 2!} + \dfrac{x^5}{5 \cdot 4!} - \cdots$

Part II: Write Out (10 points each)

Show all your work. Appropriate partial credit will be given. You may not use a calculator.

14. Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \dfrac{(x-3)^n}{5^n(n+1)}$.
(Full credit requires determining what happens at the endpoints of the interval.)

15. For what values of t is the angle between the vectors $\langle t, 1, 1 \rangle$ and $\langle t, t, -1 \rangle$ equal to $90°$?

16. Obtain an upper bound for the error in approximating each of these series by its 5th partial sum, s_5 . (You are not asked to prove that the series converge, but thinking about why that is true will help you to answer the question. Also, you may take it to be well known that xe^{-x^2} is a decreasing function for $x \geq 1$.)

(a) $$\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n^2}$$

(b) $$\sum_{n=1}^{\infty} n e^{-n^2}$$

17. Determine whether or not the series $\sum_{n=1}^{\infty} \frac{n!\,3^n}{(2n)!}$ converges, giving clear justification for your answer.

18. Let $f(x) = \sqrt{x}$.

(a) Approximate $f(x)$ by a Taylor polynomial of degree 2 centered at the point $a = 9$.

(b) Find an upper bound on the error of this approximation for all x in the interval $7 \le x \le 11$. *Hint:* The Taylor remainder inequality is

$$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1},$$

where M is an upper bound on $|f^{(n+1)}(c)|$ for all c in the interval concerned.