Name_____ ID_____ Sec___

(Print) Last, First Middle

Signature_____

MATH 152 Final Exam Spring 2001

Sections 201,202 P. Yasskin

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|---|---|
| 1-12 | /48 |
| 13 | /13 |
| 14 | /13 |
| 15 | /13 |
| 16 | /13 |

Multiple Choice: (4 points each)

**1**. Compute $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$

**a**. $e^{-2}$
**b**. $1$
**c**. $e^{1/2}$
**d**. $e^{2}$
**e**. $\infty$

**2**. Compute $\int_0^{\pi/4} \sin^2\theta\cos^2\theta\, d\theta$

**a**. $\frac{\pi}{32}$
**b**. $\frac{\pi}{16}$
**c**. $\frac{\pi}{8}$
**d**. $\frac{\pi}{4}$
**e**. $\frac{\pi}{2}$

**3**. The region below $y=\frac{1}{x}$ above the $x$-axis between $x=1$ and $x=\infty$ is rotated about the $x$-axis. Find the volume of the solid of revolution.

**a**. $\frac{\pi}{4}$
**b**. $\frac{\pi}{2}$
**c**. $\pi$
**d**. $2\pi$
**e**. $4\pi$

**4**. Compute $\int_0^1 x^2 e^{-x}\,dx$

**a**. $-5e^{-1}$
**b**. $2 - 5e^{-1}$
**c**. $-e^{-1}$
**d**. $2 - e^{-1}$
**e**. $e^{-1} - 2$

**5**. Find the average value of the function $f(x) = \dfrac{1}{(x^2+9)^{3/2}}$ on the interval $[0,3]$.

**a**. $\dfrac{\sqrt{2}}{6}$
**b**. $\dfrac{\sqrt{2}}{27}$
**c**. $\dfrac{\sqrt{2}}{9}$
**d**. $\dfrac{1}{9\sqrt{2}}$
**e**. $\dfrac{1}{27\sqrt{2}}$

**6**. Find the angle between the vector $\vec{u} = (2,1,-2)$ and the normal to the plane through $P = (3,-4,12)$ containing the vectors $\vec{v} = (1,0,0)$ and $\vec{w} = (0,-3,4)$.

**a**. $\arccos\left(\dfrac{-22}{39}\right)$
**b**. $\arccos\left(\dfrac{15}{2}\right)$
**c**. $\arccos\left(\dfrac{3}{2}\right)$
**d**. $\arccos\left(\dfrac{2}{15}\right)$
**e**. $\arccos\left(\dfrac{2}{3}\right)$

**7**. Use the 4th degree Maclaurin polynomial for $e^{-x^2}$ to estimate $\int_0^1 e^{-x^2}\,dx$.

**a**. $1 - \frac{1}{3} + \frac{1}{5}$
**b**. $1 + \frac{1}{3} + \frac{1}{5}$
**c**. $1 - \frac{1}{6} + \frac{1}{120}$
**d**. $1 + \frac{1}{3} + \frac{1}{10}$
**e**. $1 - \frac{1}{3} + \frac{1}{10}$

**8**. The series $\sum_{n=1}^{\infty} \frac{n}{n^{3/2}+1}$ is

**a**. convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
**b**. conv. by the Limit Comp. Test with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ but not by the Comp. Test
**c**. divergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
**d**. div. by the Limit Comp. Test with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ but not by the Comp. Test
**e**. none of these

**9**. The area below $y = x^2$, above the $x$-axis, between $x = 1$ and $x = 2$ is rotated about the $y$-axis. Find the volume of the solid of revolution.

**a**. $4\pi$
**b**. $\frac{15\pi}{4}$
**c**. $\frac{15\pi}{2}$
**d**. $8\pi$
**e**. $\frac{31\pi}{5}$

**10**. Compute $\sum_{n=2}^{\infty}\left(\frac{n+1}{n-1}-\frac{n+2}{n}\right)$

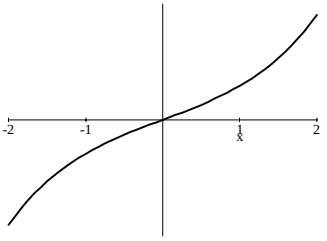
**a**. 0
**b**. 1
**c**. 2
**d**. 3
**e**. divergent

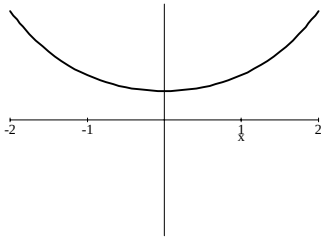
**11**. The Maclaurin series for $\sinh x$ is

$$\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

If you use the $5^{\text{th}}$-degree Maclaurin polynomial to approximate $\sinh x$ on the interval $\left[\frac{1}{2}, 2\right]$, bound the error in the approximation using the Taylor Remainder Inequality

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$ where $M \ge |f^{(n+1)}(c)|$ for all $c$ between $x$ and $a$.

HINT: $\sinh x$: 


$\cosh x$: 


**a**. $\frac{4}{15}\cosh 2$
**b**. $\frac{4}{45}\sinh 2$
**c**. $\frac{4}{45}\cosh\frac{1}{2}$
**d**. $\frac{4}{15}\sinh\frac{1}{2}$
**e**. $\frac{4}{45}\cosh 2$

**12**. Find the point $(a,b,c)$ where the line $x = 2-t \quad y = 3+2t \quad z = 4+t$ intersects the plane $2x - y + 3z = 14$. Then $a+b+c =$

**a**. 1
**b**. 3
**c**. 5
**d**. 7
**e**. 9

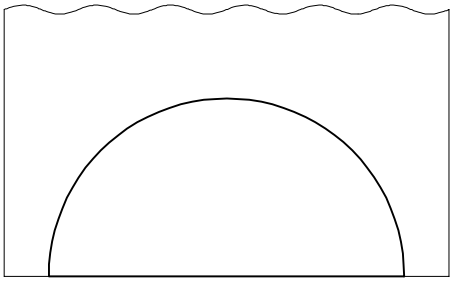
Work Out (13 points each)

Show all your work. Partial credit will be given. You may not use a calculator.

**13**. Find the solution of the differential equation $x^3 \frac{dy}{dx} - 2y = 4$ satisfying the initial condition $y(1) = 3$.

**14**. The curve $y = x^2$ for $0 \leq x \leq \sqrt{2}$ is rotated about the $y$-axis. Find the surface area of the resulting surface.

**15**. A plate in the shape of a semicircle is placed at the bottom if a tank with the straight edge down. The radius of the circle is $4$ cm and the water in the tank is $6$ cm deep.



What is the force on the plate?

(The density of water is $\rho = 1000\frac{\text{kg}}{\text{m}^3}$ and the acceleration of gravity is $g = 9.8\frac{\text{m}}{\text{sec}^2}$, but you may leave your answer in terms of $\rho g$.)

**16**. Find the interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(x-3)^n}{n(\ln n)^2}$.

Be sure to check the endpoints.
Name or quote the test(s) you use and check out all requirements of the test.