Name____________________ Sec______ ID______________

1-12	/48
13	/15+5
14	/20
15	/20
Total	/108

MATH 152 Honors Final Exam Spring 2006

Sections 201,202 P. Yasskin

Multiple Choice: (4 points each)

1. The temperature along a probe which is π m long is given by $T = 75 + \sin x$ for $0 \le x \le \pi$. Find the average temperature of the probe.

a. $75 + \frac{2}{\pi}$

b. $75 - \frac{1}{\pi}$

c. 75

d. $75\pi - 1$

e. $75\pi + 2$

2. Compute $\int_0^1 xe^{2x}\,dx$.

a. $\frac{1}{2}e^2$

b. $\frac{1}{4}e^2$

c. $\frac{1}{2}(e^2 + 1)$

d. $\frac{1}{4}(e^2 + 1)$

e. $\frac{1}{4}(3e^2 + 1)$

3. Compute $\int_1^{\sqrt{3}} \frac{2x+2}{x(1+x^2)}\,dx$

a. $\ln\sqrt{3} + \frac{\pi}{12}$

b. $\ln 3$

c. $\ln 3 + \frac{\pi}{6}$

d. $\ln \frac{3}{2}$

e. $\ln \frac{3}{2} + \frac{\pi}{6}$

4. If $y = f(x)$ is a solution of the differential equation $\frac{dy}{dx} = x + y^2$ satistfing the initial condition $f(1) = 2$, then $f'(1) =$

a. 1

b. 2

c. 3

d. 4

e. 5

5. Compute $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{16^n (2n)!}$

a. 0

b. $\frac{1}{2}$

c. $\frac{1}{\sqrt{2}}$

d. 1

e. -1

6. The region between $y = 3x - x^2$ and the x-axis is rotated about the y-axis. Find the volume of the solid swept out.

a. $\frac{81}{5}\pi$

b. $\frac{81}{10}\pi$

c. $\frac{27}{2}\pi$

d. 27π

e. $\frac{81}{2}\pi$

7. The region between $y = 3x - x^2$ and the x-axis is rotated about the x-axis. Find the volume of the solid swept out.

a. $\frac{81}{5}\pi$

b. $\frac{81}{10}\pi$

c. $\frac{27}{2}\pi$

d. 27π

e. $\frac{81}{2}\pi$

8. Find the length of the curve $y = \ln(\cos x)$ for $0 \le x \le \frac{\pi}{4}$.

a. $\ln\left(\sqrt{2}\right)$

b. $\ln\left(\frac{1}{\sqrt{2}}\right)$

c. $\frac{1}{\ln\left(\sqrt{2}\right)}$

d. $\ln\left(\sqrt{2} - 1\right)$

e. $\ln\left(\sqrt{2} + 1\right)$

9. The curve $x = t^3$, $y = t^2$ for $0 \le t \le 1$ is rotated about the x-axis. Which integral gives the area of the surface swept out?

a. $\int_0^1 \pi t^2 \sqrt{9t^4 + 4t^2}\, dt$

b. $\int_0^1 2\pi t^3 \sqrt{9t^4 + 4t^2}\, dt$

c. $\int_0^1 \pi t^3 \sqrt{9t^2 + 4}\, dt$

d. $\int_0^1 2\pi t^3 \sqrt{9t^2 + 4}\, dt$

e. $\int_0^1 2\pi t^2 \sqrt{9t^2 + 4}\, dt$

10. Compute $\int_2^6 \frac{1}{(x-2)^{3/2}}\, dx$

a. -2

b. -1

c. 1

d. 2

e. The integral diverges.

11. Compute $\lim_{x \to 0} \dfrac{\sin(x) - x}{x^2(e^x - e^{-x})}$

a. $-\infty$

b. $\dfrac{-1}{12}$

c. $\dfrac{1}{6}$

d. $\dfrac{1}{3}$

e. ∞

12. If you approximate $f(x) = \ln(1 - x)$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ by its 3^{rd} degree Maclaurin polynomial $T_3(x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3$, then the Taylor Remainder Theorem says the error is less than

Taylor Remainder Theorem:
If $T_n(x)$ is the n^{th} degree Taylor polynomial for $f(x)$ about $x = a$ then there is a number c between a and x such that the remainder is

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - a)^{(n+1)}$$

a. $\dfrac{1}{4}$

b. $\dfrac{1}{8}$

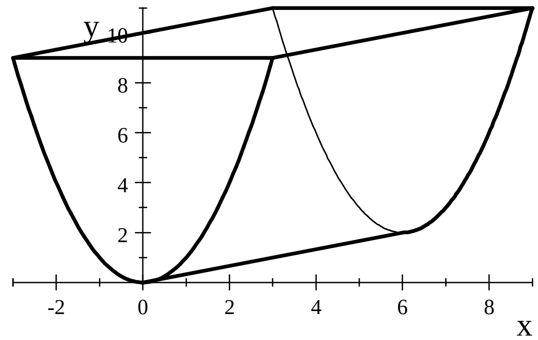
c. $\dfrac{1}{16}$

d. $\dfrac{1}{32}$

e. $\dfrac{1}{64}$

13. (15 points) A water trough is 10 ft long and has vertical ends in the shape of the paraboloid $y = x^2$ up to a height of 9 ft. It is filled with water.

Take the density of water as $\rho g = 60\ \dfrac{\text{lb}}{\text{ft}^3}$.



Do one of the following two problems. (**5 points extra credit if you do both**.)

a. Find the work done to pump the water out the top of the trough.

b. Find the force of the water on the end of the trough.

14. (20 points) Consider the sequence defined recursively by

$$a_1 = 4 \qquad \text{and} \qquad a_{n+1} = \sqrt{2a_n}$$

a. (4 pt) Write out the first 4 terms and simplify, e.g. $\sqrt{8} = 2\sqrt{2}$

$a_1 =$ __________________________ $a_2 =$ __________________________

$a_3 =$ __________________________ $a_4 =$ __________________________

b. (5 pt) Is the sequence increasing or decreasing? Prove it.

c. (5 pt) If it is increasing, find an upper bound.
If it is decreasing, find a lower bound. Prove it.

d. (2 pt) What do (b) and (c) imply?

e. (4 pt) Find the limit of the sequence.

15. (20 points) A brine solution that contains 0.4 kg of salt per liter flows at a constant rate of 2 liters per minute into a large tank that initially held 100 liters of brine that contains 0.5 kg of salt per liter. The solution in the tank is kept well mixed and flows out of the tank at 2 liters per minute. Let $S(t)$ be the amount of salt on the tank at time t.

a. (8 pt) Set up the differential equation and initial condition for $S(t)$.

b. (12 pt) How much salt is in the tank after 50 minutes?