Student (Print) ______________________________ Section ________

Last, First Middle

Student (Sign) ______________________________

Student ID ______________________________

Instructor ______________________________

MATH 152, Fall 2007

Common Exam 1

Test Form A

Solutions

Instructions:

You may not use notes, books, calculator or computer.

Part I is multiple choice. There is no partial credit.

Mark the Scantron with a #2 pencil. For your own records, also circle your choices in this exam. Scantrons will be collected after 90 minutes and may not be returned.

Part II is work out. Show all your work. Partial credit will be given.

THE AGGIE CODE OF HONOR:
An Aggie does not lie, cheat or steal, or tolerate those who do.

For Dept use Only:	
1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

Part I: Multiple Choice (5 points each)

There is no partial credit.

1. Compute $\int_0^1 xe^{(x^2+1)}\,dx$

a. $\frac{1}{2}e$
b. $\frac{1}{2}(e-1)$
c. $\frac{1}{2}e^2$
d. $\frac{1}{2}(e^2-1)$
e. $\frac{1}{2}(e^2-e)$ correct choice

Use the substitution $u = x^2+1 \quad du = 2x\,dx \quad \frac{1}{2}\,du = x\,dx$

$$\int_0^1 xe^{(x^2+1)}\,dx = \frac{1}{2}\int_1^2 e^u\,du = \left[\frac{1}{2}e^u\right]_1^2 = \frac{1}{2}(e^2-e)$$

2. Compute $\int_0^{\pi/2} x\cos x\,dx$

a. $\frac{\pi}{2}-1$ correct choice
b. $1+\frac{\pi}{2}$
c. $\frac{\pi}{2}$
d. $1-\pi$
e. $\pi-1$

Use integration by parts with $u = x \quad dv = \cos x\,dx$
$du = dx \quad v = \sin x$

$$\int_0^{\pi/2} x\cos x\,dx = \left[x\sin x - \int \sin x\,dx\right]_0^{\pi/2} = \left[x\sin x + \cos x\right]_0^{\pi/2}$$
$$= \left(\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) - (\cos 0) = \frac{\pi}{2}-1$$

3. Find the area below the parabola, $y = 3x - x^2$, above the x-axis.

a. $\frac{1}{2}$
b. $\frac{9}{2}$ correct choice
c. $\frac{27}{2}$
d. $\frac{81}{2}$
e. $\frac{10}{3}$

The parabola intersects the x-axis when $3x - x^2 = 0$ or $x = 0, 3$.

$$A = \int_0^3 (3x-x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{2} - 9 = \frac{9}{2}$$

4. Find the average value of $f(x) = e^{3x}$ on the interval $[0, 2]$.

a. $\frac{1}{3}(e^6 - 1)$

b. $\frac{1}{3}e^6$

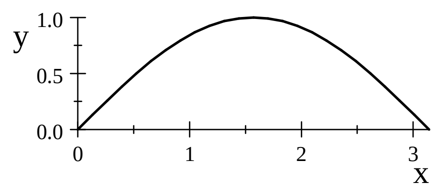
c. $\frac{1}{6}(e^6 - 1)$ correct choice

d. $\frac{1}{6}e^6$

e. $(e^6 - 1)$

$f_{\text{ave}} = \frac{1}{2}\int_0^2 e^{3x}\,dx = \left[\frac{1}{6}e^{3x}\right]_0^2 = \frac{1}{6}(e^6 - 1)$

5. The region shown at the right is bounded above by $y = \sin x$ and below by the x-axis. It is rotated about the x-axis. Find the volume swept out.



a. $\frac{\pi^2}{2}$ correct choice

b. $2\pi^2$

c. 2π

d. $\frac{\pi}{2}$

e. $\frac{\pi}{4}$

As an x-integral, a slice is vertical and rotates into a disk.

$V = \int_0^\pi \pi R^2\,dx = \int_0^\pi \pi(\sin x)^2\,dx = \pi\int_0^\pi \frac{1-\cos(2x)}{2}\,dx = \frac{\pi}{2}\left[x - \frac{\sin(2x)}{2}\right]_0^\pi = \frac{\pi^2}{2}$

6. The region in Problem 5 is rotated about the the line $x = -1$.
Which formula gives the volume swept out?

a. $\int_0^\pi \pi\left((1+\sin x)^2 - 1\right)dx$

b. $\int_0^\pi 2\pi(x+1)\sin x\,dx$ correct choice

c. $\int_0^\pi \pi(x-1)\sin x\,dx$

d. $\int_{-1}^\pi 2\pi x\sin x\,dx$

e. $\int_0^\pi \pi(1+\sin x)^2\,dx$

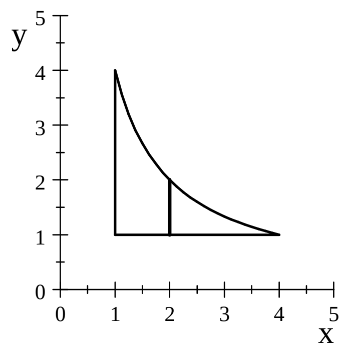
As an x-integral, a slice is vertical and rotates into a cylinder.

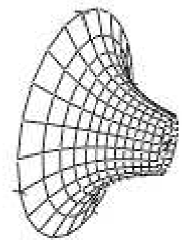
$V = \int_0^\pi 2\pi rh\,dx = \int_0^\pi 2\pi(x+1)\sin x\,dx$

This integral can be computed using integration by parts.

7. The region bounded by the curves $x = 1$, $y = 1$ and $y = \frac{4}{x}$ is rotated about the x-axis. Find the volume swept out.

a. $\pi(8\ln 4 - 15)$
b. $\pi(15 - 8\ln 4)$
c. 12π
d. 9π correct choice
e. 8π

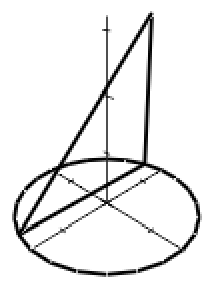




As an x-integral, a slice is vertical and rotates into a washer.

$$V = \int_1^4 (\pi R^2 - \pi r^2)\,dx = \int_1^4 \left[\pi\left(\frac{4}{x}\right)^2 - \pi(1)^2\right] dx = \pi\int_1^4 \left(\frac{16}{x^2} - 1\right) dx = \pi\left[\frac{-16}{x} - x\right]_1^4 = 9\pi$$

8. A solid has a base which is a circle of radius 2 and has cross sections perpendicular to the y-axis which are isosceles right triangles with a leg on the base. Find the volume of the solid.



a. $\frac{32}{3}$
b. $\frac{64}{3}$ correct choice
c. $\frac{128}{3}$
d. $\frac{16}{3}\pi$
e. $\frac{32}{3}\pi$

The width of the base is $W(y) = 2\sqrt{4 - y^2}$. The height is $H = W$

The area of the cross section is $A(y) = \frac{1}{2}WH = \frac{1}{2}2\sqrt{4-y^2}\,2\sqrt{4-y^2} = 2(4 - y^2)$.

$$V = \int_{-2}^{2} A(y)\,dy = \int_{-2}^{2} 2(4 - y^2)\,dy = 2\left[4y - \frac{y^3}{3}\right]_{-2}^{2} = 4\left(8 - \frac{8}{3}\right) = 32\left(\frac{2}{3}\right) = \frac{64}{3}$$

9. A certain spring is at rest when its mass is at $x = 0$.
It requires 24 Joules of work to stretch it from $x = 0$ to $x = 4$ meters.
What is the force required to maintain the mass at 4 meters?

a. 48 Newtons
b. 18 Newtons
c. 12 Newtons correct choice
d. 6 Newtons
e. 24 Newtons

The force is $F = kx$. The work is $W = \int_0^4 F\,dx = \int_0^4 kx\,dx = \left[\frac{1}{2}kx^2\right]_0^4 = 8k = 24$.

So the force constant is $k = 3$. And the force is $F = kx = 3 \cdot 4 = 12$.

10. Find the partial fraction expansion for $f(x) = \dfrac{5x^2 + x + 12}{x^3 + 4x}$.

a. $\dfrac{1}{x} + \dfrac{3x - 2}{x^2 + 4}$
b. $\dfrac{2}{x} + \dfrac{x - 3}{x^2 + 4}$
c. $\dfrac{1}{x} + \dfrac{2x + 3}{x^2 + 4}$
d. $\dfrac{2}{x} + \dfrac{3x + 1}{x^2 + 4}$
e. $\dfrac{3}{x} + \dfrac{2x + 1}{x^2 + 4}$ correct choice

$$f(x) = \frac{5x^2 + x + 12}{x^3 + 4x} = \frac{5x^2 + x + 12}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

Clear the denominator: $5x^2 + x + 12 = A(x^2 + 4) + x(Bx + C) = (A + B)x^2 + Cx + 4A$

Equate coefficients: $A + B = 5 \quad C = 1 \quad 4A = 12$

Solve: $A = 3 \quad B = 2 \quad C = 1$

Substitute back: $f(x) = \dfrac{5x^2 + x + 12}{x^3 + 4x} = \dfrac{3}{x} + \dfrac{2x + 1}{x^2 + 4}$

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

11. Compute

a. (5 points) $\int \cos^3\theta \, d\theta$

$u = \sin\theta \qquad du = \cos\theta \, d\theta$

$$\int \cos^3\theta \, d\theta = \int (1 - \sin^2\theta)\cos\theta \, d\theta = \int (1 - u^2)\, du = \left[u - \frac{u^3}{3} \right] + C = \sin\theta - \frac{\sin^3\theta}{3} + C$$

Check: $\frac{d}{d\theta}\left(\sin\theta - \frac{\sin^3\theta}{3}\right) = \cos\theta - \frac{1}{3}3\sin^2\theta(\cos\theta) = \cos\theta(1 - \sin^2\theta) = \cos^3\theta$

b. (5 points) $\int x^3 \ln x \, dx$

Use integration by parts with $u = \ln x \qquad dv = x^3\, dx$

$du = \frac{1}{x}\, dx \qquad v = \frac{x^4}{4}$

$$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\frac{1}{x}\, dx = \frac{x^4}{4}\ln x - \int \frac{x^3}{4}\, dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

Check: $\frac{d}{dx}\left(\frac{x^4}{4}\ln x - \frac{x^4}{16}\right) = x^3 \ln x + \frac{x^4}{4}\frac{1}{x} - \frac{x^3}{4} = x^3 \ln x$

12. Find the area between the cubic $y = x^3 - x^2$ and the line $y = 2x$.

The curves intersect when $x^3 - x^2 = 2x$ or $x^3 - x^2 - 2x = 0$ or $x(x^2 - x - 2) = 0$
or $x(x+1)(x-2) = 0$ or $x = -1, 0, 2$

Between -1 and 0, the cubic is above the line. (Plug in $x = -1/2$.)

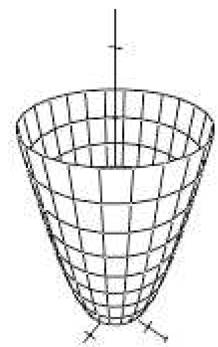
Between 0 and 2, the line is above the cubic. (Plug in $x = 1$.)

So the area is

$$A = \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx + \int_{0}^{2} (2x - x^3 + x^2)\,dx = \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right]_{-1}^{0} + \left[x^2 + \frac{x^3}{3} - \frac{x^4}{4} \right]_{0}^{2}$$

$$= (0) - \left(\frac{1}{4} - \frac{-1}{3} - 1 \right) + \left(4 + \frac{8}{3} - 4 \right) - (0) = \frac{-3 - 4 + 12 + 32}{12} = \frac{37}{12}$$

13. A water tower is made by rotating the curve $y = x^4$ about the y-axis, where x and y are in meters. If the tower is filled with water (of density $\rho = 1000$ kg/m^3) up to height $y = 25$ m, how much work is done to pump all the water out a faucet at height 30 m? Assume the acceleration of gravity is $g = 9.8$ m/sec^2. You may give your answer as a multiple of ρg.



The cross section at height y is a circle of radius $x = y^{1/4}$ an hence area $A = \pi x^2 = \pi y^{1/2}$.

A slice of thickness dy has volume $dV = A\,dy = \pi y^{1/2}\,dy$ and mass $dm = \rho\,dV = \rho\pi y^{1/2}\,dy$.

This slice must be lifted a distance $h = 30 - y$.

So the work to lift it is $dW = dm\,g\,h = \rho\pi g(30 - y)y^{1/2}\,dy$.

The total work is

$$W = \int_0^{25} \rho\pi g(30 - y)y^{1/2}\,dy = \pi\rho g \int_0^{25} (30y^{1/2} - y^{3/2})\,dy = \pi\rho g \left[30\frac{2y^{3/2}}{3} - \frac{2y^{5/2}}{5} \right]_0^{25}$$

$$= \pi\rho g \left(30\frac{2 \cdot 5^3}{3} - \frac{2 \cdot 5^5}{5} \right) = \pi\rho g(4 \cdot 5^4 - 2 \cdot 5^4) = 2 \cdot 5^4 \pi\rho g = 1250\pi\rho g$$

$$= 3.8485 \times 10^7 \text{ Joules}$$

14. Compute $\int_0^1 \frac{x^2}{(4-x^2)^{3/2}}\,dx$

Let $x = 2\sin\theta$. Then $dx = 2\cos\theta\,d\theta$ and $4 - x^2 = 4 - 4\sin^2\theta = 4\cos^2\theta$

Change limits: $x = 0$ @ $\theta = 0$ and $x = 1$ @ $\sin\theta = \frac{1}{2}$ or $\theta = \frac{\pi}{6}$

$$\int_0^1 \frac{x^2}{(4-x^2)^{3/2}}\,dx = \int_0^{\pi/6} \frac{4\sin^2\theta}{8\cos^3\theta}\,2\cos\theta\,d\theta = \int_0^{\pi/6} \tan^2\theta\,d\theta = \int_0^{\pi/6} (\sec^2\theta - 1)\,d\theta$$

$$= \Big[\tan\theta - \theta\Big]_0^{\pi/6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6}$$

15. Compute $\int_0^4 \frac{x-4}{x^2+16}\,dx$

$$\int \frac{x-4}{x^2+16}\,dx = \int \frac{x}{x^2+16}\,dx + \int \frac{-4}{x^2+16}\,dx$$

Integral 1: $u = x^2 + 16$ $du = 2x\,dx$ $\frac{1}{2}du = x\,dx$

$$\int \frac{x}{x^2+16}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+16) + C$$

Integral 2: $x = 4u$ $dx = 4du$

$$\int \frac{-4}{x^2+16}\,dx = \int \frac{-16}{16u^2+16}\,du = -\arctan(u) + C = -\arctan\left(\frac{x}{4}\right) + C$$

So $\int \frac{x-4}{x^2+16}\,dx = \frac{1}{2}\ln(x^2+16) - \arctan\left(\frac{x}{4}\right) + C$

So $\int_0^4 \frac{x-4}{x^2+16}\,dx = \left(\frac{1}{2}\ln 32 - \arctan(1)\right) - \left(\frac{1}{2}\ln 16 - \arctan(0)\right)$

$$= \frac{1}{2}\ln 2 - \frac{\pi}{4}$$