Name________________________ Sec______ ID______________

1-11	/66
12	/12
13	/15
14	/10
Total	/103

MATH 152 Honors   Final Exam   Fall 2007

Sections 201,202   P. Yasskin

Multiple Choice: (6 points each)

1. A 2 meter bar has linear density $\rho = 1 + x^3$ kg/m where x is measured from one end. Find the average density of the bar.

a. 2 kg/m

b. 3 kg/m

c. 4.5 kg/m

d. 5 kg/m

e. 6 kg/m

2. A 2 meter bar has linear density $\rho = 1 + x^3$ kg/m where x is measured from one end. Find the center of mass of the bar.

a. $\frac{5}{7}$ m

b. $\frac{5}{6}$ m

c. $\frac{6}{5}$ m

d. $\frac{7}{5}$ m

e. $\frac{42}{5}$ m

3. Compute $\int x \arctan x \, dx$.

a. $\dfrac{3x^2}{2} \arctan x - \dfrac{x}{2} \ln(x^2+1) + C$

b. $\dfrac{3x^2}{2} \arctan x + \dfrac{x}{2} \ln(x^2+1) + C$

c. $\dfrac{x^2}{2} \arctan x + \dfrac{3x}{2} \ln(x^2+1) + C$

d. $\dfrac{x^2}{2} \arctan x - \dfrac{1}{2} x - \dfrac{1}{2} \arctan x + C$

e. $\dfrac{x^2}{2} \arctan x - \dfrac{1}{2} x + \dfrac{1}{2} \arctan x + C$

4. Find the arclength of the parametric curve $x = t^4$ $y = \dfrac{1}{2} t^6$ for $0 \le t \le 1$.

a. $\dfrac{61}{54}$

b. $\dfrac{16}{9}$

c. $\dfrac{11}{9}$

d. $\dfrac{1}{9}$

e. $\dfrac{1}{54}$

5. Which term appears in the partial fraction expansion of $\dfrac{4x^2 - 2x + 2}{(x-1)^2(x^2+1)}$?

a. $\dfrac{-2}{(x-1)^2}$

b. $\dfrac{1}{(x-1)^2}$

c. $\dfrac{2}{(x-1)^2}$

d. $\dfrac{-2}{x-1}$

e. $\dfrac{2}{x-1}$

6. The base of a solid is the region bounded by the curves $y = x^2$, $y = -x^2$ and $x = 2$. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

a. $\dfrac{128}{5}$

b. $\dfrac{32}{5}$

c. $\dfrac{16}{3}$

d. $\dfrac{8}{3}$

e. $\dfrac{16}{15}$

7. Find the solution of the differential equation $\frac{dy}{dx} = xy^2 + x$ satisfying the initial condition $y(0) = 1$.

a. $y = \sin\left(\frac{x^2}{2} + \frac{\pi}{2}\right)$

b. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{4}\right)$

c. $y = \sin\left(\frac{x^2}{2}\right) + 1$

d. $y = \tan\left(\frac{x^2}{2}\right) + 1$

e. $y = \cos\left(\frac{x^2}{2}\right)$

8. If $g(x) = \cos(x^2)$, what is $g^{(8)}(0)$, the 8th derivative at zero?
HINT: What is the coefficient of x^8 in the Maclaurin series for $\cos(x^2)$?

a. $8 \cdot 7 \cdot 6 \cdot 5$

b. $\frac{1}{8 \cdot 7 \cdot 6 \cdot 5}$

c. $4!$

d. $\frac{1}{4!}$

e. $\frac{1}{8!}$

9. Suppose the series $\sum_{n=1}^{\infty} n e^{(-n^2)}$ is approximated by its 9^{th} partial sum $\sum_{n=1}^{9} n e^{(-n^2)}$.

Use an integral to bound the error in this approximation.

a. $\frac{1}{2}e^{-64}$

b. $\frac{1}{2}e^{-81}$

c. $\frac{1}{2}e^{-100}$

d. $\frac{1}{2}e^{-121}$

e. $\frac{1}{2}e^{-144}$

10. Find the area of the triangle with vertices $P = (2,-1,3)$, $Q = (1,2,1)$ and $R = (3,1,4)$.

a. $\frac{1}{2}\sqrt{73}$

b. $\sqrt{73}$

c. $\frac{5}{2}\sqrt{3}$

d. $5\sqrt{3}$

e. $-5\sqrt{3}$

11. If $\vec{u}$ points North East and $\vec{v}$ points West, in which direction does $\vec{u}\times\vec{v}$ point?

a. North West

b. South

c. South East

d. Up

e. Down

Work Out: (Points indicated. Part credit possible.)

12. (12 points) Compute $\int_2^4 \frac{8}{x^3\sqrt{x^2-4}}\,dx$

13. (15 points) The curve $y = x^2$ is rotated about the y-axis to form a bowl. If the bowl contains 8π cm^3 of water, what is the height of the water in the bowl?

14. (10 points) This question is designed to teach you about infinite products.

a. (2 pt) Define the partial sum, S_k, and the sum, S, of an infinite series $\sum_{n=0}^{\infty} a_n$.

(Give one sentence including one equation for each.)

b. (2 pt) By analogy, define the partial product, P_k, and the product, P, of an infinite product $\prod_{n=0}^{\infty} a_n$. (Give one sentence including one equation for each.)

c. (4 pt) Compute $\prod_{n=0}^{\infty}\left(1+x^{(2^n)}\right)$.

HINT: Multiply out the first 3 partial products P_0, P_1 and P_2. Then find P_k and P.

d. (2 pt) For which x does this infinite product converge? To what function?