Name________________________ Sec______ ID______________

1-11	/55
12	/12
13	/22
14	/16
Total	/105

MATH 152 Honors — Final Exam — Fall 2008

Sections 201,202 — P. Yasskin

Multiple Choice: (5 points each)

1. Find the average value of $f(x) = \sin x$ on the interval $0 \le x \le \pi$.

a. $\dfrac{1}{\pi}$

b. $\dfrac{2}{\pi}$

c. $\dfrac{3}{\pi}$

d. $\dfrac{1}{2}$

e. $\dfrac{1}{3}$

2. Compute $\displaystyle\int_0^3 \frac{1}{(25 - x^2)^{3/2}}\,dx$.

a. $\dfrac{4}{15}$

b. $\dfrac{4}{75}$

c. $\dfrac{3}{20}$

d. $\dfrac{3}{100}$

e. $\dfrac{3}{500}$

3. The region below $y = x^2$ for $0 \le x \le 2$ is rotated about the y-axis. Find the volume of the solid swept out.

a. 8π

b. 4π

c. $\frac{32\pi}{5}$

d. $\frac{32\pi}{3}$

e. $\frac{16\pi}{3}$

4. Compute $\int \frac{x+2}{x^3+x}\,dx$. HINT: Equate coefficients.

a. $\ln \frac{x^2}{(x^2+1)^2} + \arctan(x) + C$

b. $\ln \frac{x^2}{(x^2+1)^2} - \arctan(x) + C$

c. $\ln\left|\frac{x}{x^2+1}\right| + \arctan(x) + C$

d. $\ln\left|\frac{x}{x^2+1}\right| - \arctan(x) + C$

e. $\ln \frac{x^2}{x^2+1} + \arctan(x) + C$

5. Which of the following differential equations is NOT separable?

a. $\frac{dy}{dx} = x + xy$

b. $\frac{dy}{dx} = xy + xy^2 + x^2y + x^2y^2$

c. $\frac{dy}{dx} = \frac{1}{xy} + \frac{1}{x} + \frac{1}{y}$

d. $\frac{dy}{dx} = \frac{x}{y} + x + \frac{1}{y} + 1$

e. $\frac{dy}{dx} = \frac{y}{x} - y$

6. Suppose $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} = xy^2 + x^2$ satisfying the initial condition $f(1) = 2$. Find $f'(1)$.

a. 1
b. 2
c. 3
d. 4
e. 5

7. Compute $\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$.

a. e
b. $1 - e$
c. $e - e^2$
d. $e - 1$
e. $e^2 - e$

8. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n}$

a. Converges by the n^{th}-Term Divergence Test.

b. Diverges by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

c. Diverges by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

d. Diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

e. Diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

9. The series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

a. Converges by the Ratio Test.

b. Diverges by the Integral Test.

c. Converges because it is a p-series with $p = 2 > 1$.

d. Converges because it is a p-series with $p = 1$.

e. Diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

10. Compute $\lim_{x \to 0} \frac{\sin(x^2) - x^2}{x^5}$

a. $-\frac{1}{3}$

b. $-\frac{1}{6}$

c. 0

d. $\frac{1}{6}$

e. $\frac{1}{3}$

11. Find the volume of the parallelepiped with edge vectors

$$\vec{a} = (2,1,4), \quad \vec{b} = (0,3,2) \quad \text{and} \quad \vec{c} = (-2,1,3).$$

a. 42

b. 34

c. 28

d. 21

e. 17

Work Out: (Points indicated. Part credit possible.)

12. (12 points) The curve $x = 2t^2$, $y = t^3$ for $0 \leq t \leq 1$ is rotated about the y-axis. Find the surface area of the surface swept out. Don't simplify your numbers.

13. (22 points) A bucket starts out containing 4 liters of salt water with concentration of 60 gm of salt per liter. Salt water is added to the bucket at 3 liters per minute with concentration of 50 gm of salt per liter. The water in the bucket is kept mixed but is leaking out at the rate of 1 liter per minute. (Note the amount of water in the bucket is **not** constant.)

Let $S(t)$ denote the amount of salt in the bucket at time t.

a. (7 pt) Write a differential equation for $S(t)$ and an initial condition for $S(t)$.
HINT: How much water is in the bucket at time t?

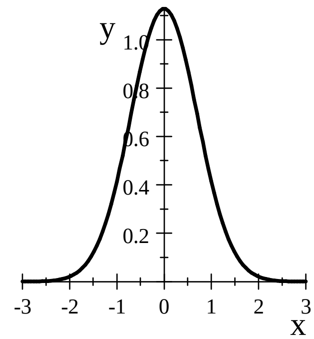
b. (2 pt) Select **one**: The differential equation is

☐ separable ☐ linear ☐ both ☐ neither

c. (9 pt) Solve the initial value problem for $S(t)$.

d. (4 pt) How much salt is in the bucket after 6 minutes?
What is the concentration after 6 minutes?

14. (16 points) The "Gaussian bell curve" used in statistics is $\frac{2}{\sqrt{\pi}} e^{-t^2}$ as shown in the graph. The "error function" is its integral:



$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

which gives the probability that the value of t is between 0 and x. For this problem we will ignore the coefficient and study the function

$$f(x) = \int_0^x e^{-t^2}\,dt$$

a. (6 pt) Find a Maclaurin series for $f(x)$ in summation notation.

b. (6 pt) Use the first 3 terms of the Maclaurin series for $f(x)$ to approximate $f(1)$. $(n = 0, 1, 2)$ Do not simplify. Find a bound on the error in this approximation. Explain why.

c. (4 pt) How many terms of the Maclaurin series for $f(x)$ would you need to approximate $f(1)$ to within 10^{-3}? Why?