

**MATH 152, FALL SEMESTER 2008
COMMON EXAMINATION I - VERSION B**

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–10), mark your responses on your ScanTron form using a No. 2 pencil. *For your own record, mark your choices on the exam as well.*
2. Calculators **should not be used** throughout the examination.
3. In Part 2 (Problems 11–14), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

<u>QN</u>	<u>PTS</u>
1–10	_____
11	_____
12	_____
13	_____
14	_____
TOTAL	

Part 1 – Multiple Choice (50 points)

Read each question carefully; each problem is worth **5 points**. Mark your responses on the ScanTron form and on the exam itself.

1. Evaluate $\int_1^e \frac{\sqrt{\ln x}}{x} dx$.

(a) $2/3$

(b) $3/2$

(c) $-\infty$

(d) $1/2$

(e) $\frac{2(e^{3/2} - 1)}{3}$

2. Evaluate $\int_0^1 x(2x - 1)^7 dx$.

(a) $2/9$

(b) $1/9$

(c) $-1/72$

(d) $1/18$

(e) $17/72$

3. Compute the area under the curve $y = xe^{-x}$, between $x = 0$ and $x = 1$.

(a) $1 - 2e^{-1}$

(b) $1 + 2e^{-1}$

(c) $\frac{1 - e^{-1}}{2}$

(d) $\frac{e^{-1} - 1}{2}$

(e) $2e^{-1} - 1$

4. Compute the area enclosed by the parabola $y = x^2$ and the straight line $y = 2x$.
- (a) $1/2$
 - (b) $2/3$
 - (c) $3/4$
 - (d) 1
 - (e) $4/3$
5. A force of 10 lb is required to hold a spring stretched $1/3$ ft beyond its natural length. How much work is done in stretching it from its natural length to $1/2$ ft beyond its natural length? (Recall and use *Hooke's Law*: the force required to maintain a spring attached x units beyond its natural length is given by $f(x) = kx$, where k is a constant.)
- (a) $15/4$ ft-lb
 - (b) $15/2$ ft-lb
 - (c) 15 ft-lb
 - (d) $45/2$ ft-lb
 - (e) 45 ft-lb
6. Use the trigonometric identity $2 \sin A \cos B \equiv \sin(A+B) + \sin(A-B)$ to compute the indefinite integral $\int \sin 3t \cos 2t \, dt$.
- (a) $-\frac{\cos 5t}{10} - \frac{\cos t}{2} + C$
 - (b) $\frac{\cos 5t}{5} + \sin t + C$
 - (c) $\frac{\cos 5t}{10} + \frac{\sin t}{2} + C$
 - (d) $-\frac{\sin 5t}{10} - \frac{\sin t}{2} + C$
 - (e) $-\frac{\cos 5t}{5} - \cos t + C$

7. Compute $\int \tan^3 \theta \sec \theta d\theta$.

(a) $\frac{\tan^3 \theta}{3} - \tan \theta + C$

(b) $\frac{\tan^3 \theta}{3} + C$

(c) $\frac{\sec^3 \theta}{3} - \sec \theta + C$

(d) $\left(\frac{\tan^4 \theta}{4}\right) \ln |\sec \theta + \tan \theta| + C$

(e) $\tan^4 \theta \sec \theta + 3 \tan^2 \theta \sec^3 \theta + C$

8. Compute $\int \frac{dx}{(3-x)(3+x)}$.

(a) $\ln |9 - x^2| + C$

(b) $\ln \left| \frac{x-3}{x+3} \right| + C$

(c) $-\frac{1}{3} \ln \left| \frac{x-3}{x+3} \right| + C$

(d) $-\frac{1}{6} \ln \left| \frac{x-3}{x+3} \right| + C$

(e) $(\ln |3-x|)(\ln |3+x|) + C$

9. Which of the following is the correct partial-fraction decomposition for the rational function

$$\frac{x^4 + x + 1}{(x^2 - 1)^2(x^2 + 1)^2}?$$

(a) $\frac{A}{x^2 - 1} + \frac{B}{x^2 - 1} + \frac{C}{x^2 + 1} + \frac{D}{x^2 + 1}$

(b) $\frac{A}{x^2 - 1} + \frac{B}{(x^2 - 1)^2} + \frac{C}{x^2 + 1} + \frac{D}{(x^2 + 1)^2}$

(c) $\frac{A}{(x-1)^2} + \frac{B}{(x+1)^2} + \frac{Cx + D}{x^2 + 1} + \frac{Ex + F}{(x^2 + 1)^2}$

(d) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{x^2 + 1} + \frac{F}{(x^2 + 1)^2}$

(e) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{Ex + F}{x^2 + 1} + \frac{Gx + H}{(x^2 + 1)^2}$

10. Suppose that f is a continuous function defined on $[0, \infty)$, and that the average value of f over the interval $[0, t]$ equals $t^2 + 1$ for every $t > 0$. Find f .

(a) $f(x) = x^2 + 1$

(b) $f(x) = 2x$

(c) $f(x) = \frac{x^3}{3} + x$

(d) $f(x) = x^3 + x$

(e) $f(x) = 3x^2 + 1$

Part 2 (55 points)

Present your solutions to the following problems (11–14) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

- 11. (7 points)** Let $a > 0$, and let \mathcal{R} be the region enclosed by $y = 2x^3$, the x -axis, and the line $x = a$. Let \mathcal{S} denote the solid obtained by rotating \mathcal{R} about the x -axis. Ascertain the value of a for which the volume of \mathcal{S} is π times the area of \mathcal{R} .

12. (7 points) Let \mathcal{S} be the solid whose base is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 2)$, and whose cross sections perpendicular to the y -axis are semicircles. Compute the volume of \mathcal{S} .

13. (i) (7 points) Let \mathcal{R}_t denote the region enclosed by the y -axis, the line $y = 1$, and the curve $y = \sqrt{x}$. Use the method of discs to compute the volume of the solid obtained by rotating the region \mathcal{R}_t about the y -axis.

(ii) (7 points) Let \mathcal{R}_b denote the region enclosed by the x -axis, the line $x = 1$, and the curve $y = \sqrt{x}$. Use the method of cylindrical shells to calculate the volume of the solid obtained by rotating the region \mathcal{R}_b about the line $x = 1$.

14. Compute each of the following integrals:

(i) (9 points) $\int \frac{(x-1)^2}{x^3+x} dx$

(ii) (9 points) $\int 4x \tan^{-1}(2x) dx$

(iii) (9 points) $\int x\sqrt{1-9x^2} dx$