

**MATH 152, FALL SEMESTER 2008  
COMMON EXAMINATION III - VERSION B**

Name (print): \_\_\_\_\_

Signature: \_\_\_\_\_

Instructor's name: \_\_\_\_\_

Section No: \_\_\_\_\_

**INSTRUCTIONS**

1. In Part 1 (Problems 1–10), mark your responses on your ScanTron form using a No. 2 pencil. *For your own record, mark your choices on the exam as well.*
2. Calculators **should not be used** throughout the examination.
3. In Part 2 (Problems 11–15), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

**Part 1 – Multiple Choice (50 points)**

Each question is worth **5 points**. Mark your responses on the ScanTron form and on the exam itself.

1. Suppose that  $\{a_n\}$  and  $\{b_n\}$  are sequences of real numbers. Given that  $\lim_{n \rightarrow \infty} a_n = 2$  and  $\lim_{n \rightarrow \infty} b_n = -3$ , determine  $\lim_{n \rightarrow \infty} (a_n^2 + 3b_n)$ .
  - (a)  $-5$
  - (b)  $13$
  - (c)  $0$
  - (d)  $11$
  - (e)  $-7$
  
2. Compute  $\lim_{n \rightarrow \infty} \frac{n}{\ln n}$ .
  - (a)  $-\infty$
  - (b)  $-1$
  - (c)  $0$
  - (d)  $1$
  - (e)  $+\infty$
  
3. Determine the radius of the sphere given by the equation  $x^2 - 2x + y^2 + z^2 - 2 = 0$ .
  - (a)  $1$
  - (b)  $2$
  - (c)  $3$
  - (d)  $\sqrt{2}$
  - (e)  $\sqrt{3}$

4. Suppose that  $0 \leq a_n \leq b_n$  for every positive integer  $n$ . Which of the following statements is always true?

(a) If  $\sum_{n=1}^{\infty} a_n$  is convergent, then so is  $\sum_{n=1}^{\infty} b_n$ .

(b) If  $\sum_{n=1}^{\infty} b_n$  is convergent, then so is  $\sum_{n=1}^{\infty} a_n$ .

(c) If  $\sum_{n=1}^{\infty} b_n$  is divergent, then so is  $\sum_{n=1}^{\infty} a_n$ .

(d) If  $\lim_{n \rightarrow \infty} b_n = 0$ , then  $\sum_{n=1}^{\infty} a_n$  is convergent.

(e) If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\lim_{n \rightarrow \infty} b_n = 0$ .

5. Consider the infinite series  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ . Which of the following statements is true?

(a) The Test for Divergence shows that the series is divergent.

(b) The Alternating Series Test shows that the series is convergent.

(c) The Ratio Test shows that the series diverges.

(d) The Ratio Test shows that the series converges.

(e) The series is absolutely convergent, hence convergent.

6. Suppose that  $\{b_n\}$  is a sequence of positive numbers, and that  $\lim_{n \rightarrow \infty} nb_n = 1$ . Which of the following statements is true? (*Hint:  $nb_n = \frac{b_n}{1/n}$* )

(a) The Ratio Test shows that the series  $\sum_{n=1}^{\infty} b_n$  is convergent.

(b) The Ratio Test shows that the series  $\sum_{n=1}^{\infty} b_n$  is divergent.

(c) The Limit Comparison Test shows that the series  $\sum_{n=1}^{\infty} b_n$  is convergent.

(d) The Limit Comparison Test shows that the series  $\sum_{n=1}^{\infty} b_n$  is divergent.

(e) The Test for Divergence shows that the series  $\sum_{n=1}^{\infty} b_n$  is divergent.

7. Compute the sum of the infinite series  $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)}$ . (*Hint*: Partial fractions)

- (a) 1/5
- (b) 1/4
- (c) 1/3
- (d) 1
- (e) 12

8. Consider the following pair of infinite series:

$$(I) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{4/3}} \quad (II) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{3/4}}$$

Which of the following statements is true?

- (a) Both series are absolutely convergent.
- (b) (I) is absolutely convergent; (II) is convergent, but not absolutely convergent.
- (c) (II) is absolutely convergent; (I) is convergent, but not absolutely convergent.
- (d) (I) is convergent, but not absolutely convergent; (II) is convergent, but not absolutely convergent.
- (e) Both series are divergent.

9. Compute the sum of the infinite series  $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$ .

- (a) 2/3
- (b) 1
- (c) 2
- (d) 4
- (e) 6

10. Which of the following is the Maclaurin series expansion of the function  $f(x) = \cos(x^2)$ ?

- (a)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$
- (b)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$
- (c)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+2}$
- (d)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$
- (e)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n}$

**Part 2 (55 points)**

*Present your solutions to the following problems (11–15) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

11. Let  $T$  denote the triangle with vertices at  $A(0, 1, 1)$ ,  $B(2, 2, -1)$ , and  $C(3, 2, 0)$ .
- (i) (5 points) Show that the sides  $AB$  and  $BC$  are perpendicular to each other.

- (ii) (5 points) Calculate the area of  $T$ .

12. (6 points) Compute the sum of the infinite series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ .

13. (10 points) Find the 3-rd degree Taylor polynomial of  $f(x) = 2x^4 + x - 1$ , at the point  $a = -1$ .

14. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n \sqrt{n}}.$$

(i) (8 points) Compute the radius of convergence of the power series.

(ii) (6 points) Determine the interval of convergence of the power series. Explain your reasoning concisely and completely.



15. Let

$$S(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} x^{2n}, \quad -\infty < x < \infty.$$

(i) (5 points) Use the series given above to express  $\int S(x) dx$  as a power series.

(ii) (5 points) Use the series obtained in (i) to express the definite integral  $\int_0^{1/2} S(x) dx$  as the sum of an infinite series.

(iii) (5 points) Suppose that the definite integral in (ii) is approximated by the sum of the first 3 terms in the infinite series in (ii). Estimate the error of this approximation. Justify your answer.

**QN** \_\_\_\_\_ **PTS**

1-10 \_\_\_\_\_

11 \_\_\_\_\_

12 \_\_\_\_\_

13 \_\_\_\_\_

14 \_\_\_\_\_

15 \_\_\_\_\_

**TOTAL**