MATH 152, SPRING 2012
HONORS EXAM II

Last Name: ______________________________ **First Name**: ______________________________

Signature: ______________________________ **Section No**: _______________

PART I: **Multiple Choice** (**4 pts each**)

1. Compute $\displaystyle\int_{-1}^{\infty} \frac{dx}{1+x^2}$.

a. $\frac{3\pi}{4}$
b. $\frac{\pi}{2}$
c. $\frac{\pi}{4}$
d. ∞
e. 0

2. The improper integral $\displaystyle\int_{1}^{e} \frac{dx}{x\ln x}$

a. diverges to ∞.
b. diverges to $-\infty$.
c. converges to 1.
d. converges to -1.
e. converges to $\frac{1}{e} - 1$.

3. $\displaystyle\int \frac{1}{x^2(x-1)}\,dx =$

a. $\ln|x-1| + \frac{1}{x} + C$
b. $\ln|x^2(x-1)| + C$
c. $\ln|x| - \frac{1}{x} - \ln|x-1| + C$
d. $-\ln|x| + \frac{1}{x} + \ln|x-1| + C$
e. $\ln|x-1| - \frac{1}{x} + C$

4. By substituting $x = 3\tan\theta$, the integral $\int_0^3 x^2\sqrt{x^2+9}\,dx$ becomes

a. $\int_0^{\pi/4} 27\tan^2\theta\sec\theta\,d\theta$
b. $\int_0^{3} 27\tan^2\theta\sec^3\theta\,d\theta$
c. $\int_0^{\pi/4} 81\tan^3\theta\sec^2\theta\,d\theta$
d. $\int_0^{\pi/4} 81\tan^2\theta\sec^2\theta\,d\theta$
e. $\int_0^{\pi/4} 81\tan^2\theta\sec^3\theta\,d\theta$

5. Find the length of the curve $x = t^2$, $y = t^3$, for $0 \le t \le 1$.

a. $\frac{1}{27}\left(13\sqrt{13} - 8\right)$
b. $\frac{2\pi}{27}\left(13\sqrt{13} - 8\right)$
c. $\frac{1}{27}\left(13\sqrt{13} - 1\right)$
d. $\frac{1}{27}$
e. $\frac{2\pi}{27}\left(13\sqrt{13} - 1\right)$

6. Which of the following integrals gives the surface area obtained by rotating the curve $y = e^{-4x}$, for $0 \le x \le 1$, about the y-axis?

a. $\int_0^1 2\pi e^{-4x}\sqrt{1+16e^{-8x}}\,dx$
b. $\int_0^1 2\pi x\sqrt{1+16e^{-8x}}\,dx$
c. $\int_1^{e^{-4}} 2\pi y\sqrt{1+\frac{1}{16y^2}}\,dy$
d. $\int_0^1 \frac{\pi}{2}\sqrt{16y^2+1}\,dy$
e. $\int_0^1 \frac{\pi}{8}\frac{\ln y}{y}\sqrt{16y^2+1}\,dy$

7. Which of the following statements is true regarding the improper integral $\int_1^{\infty} \frac{dx}{e^x + \sqrt{x}}$?

a. The integral converges because $\int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} < \int_1^{\infty} \frac{dx}{\sqrt{x}}$ and $\int_1^{\infty} \frac{dx}{\sqrt{x}}$ converges.

b. The integral diverges because $\int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} > \int_1^{\infty} \frac{dx}{\sqrt{x}}$ and $\int_1^{\infty} \frac{dx}{\sqrt{x}}$ diverges.

c. The integral diverges because $\int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} > \int_1^{\infty} \frac{dx}{e^x}$ and $\int_1^{\infty} \frac{dx}{e^x}$ diverges.

d. The integral converges because $\int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} < \int_1^{\infty} \frac{dx}{e^x}$ and $\int_1^{\infty} \frac{dx}{e^x}$ converges.

e. The integral converges to 0.

8. Find the integrating factor for the differential equation $(1 + x^2)y' = 2xy + 1 + x^2$.

a. $I(x) = 1 + x^2$

b. $I(x) = \frac{1}{1 + x^2}$

c. $I(x) = 2(1 + x^2)$

d. $I(x) = \frac{2}{1 + x^2}$

e. The equation is not linear, so there is no integrating factor.

9. If $y = f(x)$ is a solution of the Initial Value Problem: $\frac{dy}{dx} = x + y^2$ with $f(1) = 2$, then $f'(1) =$

a. 1

b. 2

c. 3

d. 4

e. 5

10. Solve the Initial Value Problem: $\dfrac{dy}{dx} = \dfrac{y+1}{x+1}$ with $y(0) = -2$. Then $y(1) =$

a. -1
b. -2
c. -3
d. -4
e. -5

11. The plot at the right is the direction field for which differential equation?

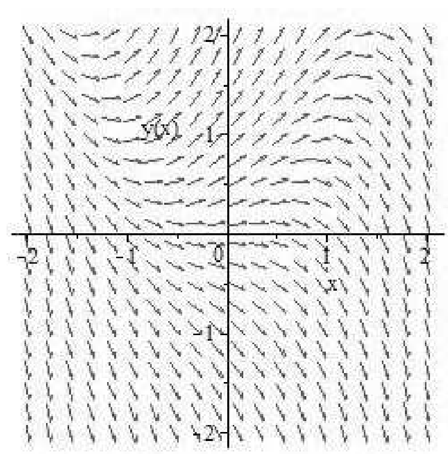
a. $\dfrac{dy}{dx} = y - x^2$

b. $\dfrac{dy}{dx} = y + x^2$

c. $\dfrac{dy}{dx} = x - y^2$

d. $\dfrac{dy}{dx} = x + y^2$

e. $\dfrac{dy}{dx} = y^2 - x$



PART II: **WORK OUT** (**10 pts each**)

Show all your work neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

12. Find the surface area obtained by rotating the curve $y = \dfrac{x^2}{4} - \dfrac{1}{2}\ln x$, for $1 \le x \le 2$, about the y-axis.

13. Integrate $\int \sqrt{16 - 9x^2}\, dx$.

14. Integrate $\int \dfrac{4x^2 - 1}{(x^2 + 1)(x - 2)}\, dx$.

15. When a dead body was found by the police its temperature was 85°F. After 2 hours its temperature dropped to 75°F. How many hours before the body was found was its temperature 95° F, assuming the room temperature was a constant 65° F?

16. A bathtub starts out with 10 gallons of water containing 5 pounds of bath salts. Pure water is entering the bathtub at 0.2 gallons per minute and bath salts are poured in at 3 pound per minute. The water is kept well mixed and drains out at 0.2 gallons per minute. How long will it take until there are 10 pounds of bath salts in the bathtub?

17. A recursive sequence is defined by $a_1 = 2, \quad a_{n+1} = 5 - \dfrac{4}{a_n}$.

a. Use Mathematical Induction to prove:
The sequence is increasing and bounded between 0 and 5, i.e.
$P(n): \quad a_{n+1} > a_n \quad \text{and} \quad 0 < a_n < 5.$

i. Initialization Step: Find some terms and verify $P(1)$ and $P(2)$ are true:

$a_1 =$______________, $a_2 =$______________, $a_3 =$______________,

$P(1):$

$P(2):$

ii. Write out $P(k)$ and $P(k+1)$:

$P(k):$

$P(k+1):$

iii. Induction Step: Assuming $P(k)$ prove $P(k+1)$:

b. What Theorem guarantees the sequence converges? Find the limit.

$\lim_{n \to \infty} a_n =$______________

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Question	Points Awarded	Points
1-11		44
12		10
13		10
14		10
15		10
16		10
17		10
Total		104