MATH 152, SPRING 2012
COMMON EXAM III - VERSION A

Last Name: ______________________________ **First Name**: ______________________________

Signature: ______________________________ **Section No**: ______________

PART I: Multiple Choice (4 pts each)

1. What is the intersection of the sphere $(x-2)^2 + (y-1)^2 + (z-3)^2 = 16$ with the xy-plane?

   (a) The circle $(x-2)^2 + (y-1)^2 = 16$.
   (b) The circle $(x-2)^2 + (y-1)^2 = 1$.
   (c) The point $(2, 1, 0)$.
   (d) The circle $(x-2)^2 + (y-1)^2 = 7$.
   (e) The sphere does not intersect the xy-plane.

2. If we apply the Ratio Test to the series $\sum_{n=1}^{\infty} \frac{(-4)^n n^2}{5^{n+1}}$, which of the following statements is true?

   (a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = -\frac{4}{5} < 1$, therefore the series converges.
   (b) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{4}{5} < 1$, therefore the series converges.
   (c) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{4}{25} < 1$, therefore the series converges.
   (d) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty > 1$, therefore the series diverges.
   (e) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$, therefore the series converges.

3. What does it mean to say $\sum_{n=1}^{\infty} a_n$ is convergent?

   (a) $\lim_{n\to\infty} a_n = 0$.
   (b) $\lim_{n\to\infty} a_n$ exists.
   (c) There are numbers A and A' such that $A \le a_n \le A'$ for all n.
   (d) There are numbers B and B' such that $B \le a_1 + a_2 + ... + a_n \le B'$ for all n.
   (e) $\lim_{n\to\infty} (a_1 + a_2 + ... + a_n)$ exists.

4. Which of the following is a unit vector in the direction of $\mathbf{b} - 3\mathbf{a}$, where $\mathbf{a} = \langle 1, -2, 2\rangle$ and $\mathbf{b} = \langle 5, 0, -3\rangle$?

(a) $\left\langle -\frac{2}{11}, -\frac{6}{11}, \frac{9}{11} \right\rangle$

(b) $\left\langle \frac{2}{11}, \frac{6}{11}, -\frac{9}{11} \right\rangle$

(c) $\langle 2, 6, -9\rangle$

(d) $\left\langle -\frac{14}{\sqrt{524}}, -\frac{2}{\sqrt{524}}, \frac{18}{\sqrt{524}} \right\rangle$

(e) $\left\langle \frac{14}{\sqrt{524}}, \frac{2}{\sqrt{524}}, -\frac{18}{\sqrt{524}} \right\rangle$

5. Represent $\frac{1}{1+4x^2}$ as a power series about 0 and give the interval of convergence.

(a) $\frac{1}{1+4x^2} = \sum_{n=0}^{\infty} (-4)^n x^{2n}$, where $|x| < \frac{1}{2}$.

(b) $\frac{1}{1+4x^2} = \sum_{n=0}^{\infty} (-4)^n x^{2n}$, where $|x| < \frac{1}{4}$.

(c) $\frac{1}{1+4x^2} = \sum_{n=0}^{\infty} (-4)^n x^{2n}$, where $|x| < 4$.

(d) $\frac{1}{1+4x^2} = \sum_{n=0}^{\infty} 4^n x^{2n}$, where $|x| < \frac{1}{2}$.

(e) $\frac{1}{1+4x^2} = \sum_{n=0}^{\infty} 4^n x^{2n}$, where $|x| < \frac{1}{4}$.

6. Find the cosine of the angle, θ, between the vectors $\mathbf{a} = \langle 2, 0, 1\rangle$ and $\mathbf{b} = \langle 1, -2, 1\rangle$.

(a) $\cos(\theta) = \frac{4}{\sqrt{30}}$

(b) $\cos(\theta) = \frac{5}{\sqrt{30}}$

(c) $\cos(\theta) = \frac{3}{\sqrt{12}}$

(d) $\cos(\theta) = \frac{5}{\sqrt{12}}$

(e) $\cos(\theta) = \frac{3}{\sqrt{30}}$

7. Find the 15th derivative at $x = 0$ for $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{7^n(n+5)} x^n$.

(a) $f^{(15)}(0) = 0$

(b) $f^{(15)}(0) = \frac{15!}{7^{15}(20)}$

(c) $f^{(15)}(0) = -\frac{15!}{7^{15}(20)}$

(d) $f^{(15)}(0) = -\frac{15}{7^{15}(20)}$

(e) $f^{(15)}(0) = \frac{15}{7^{15}(20)}$

8. Find the Taylor Polynomial of degree 2, $T_2(x)$, for $f(x) = \sqrt{x}$ centered at $a = 9$.

(a) $T_2(x) = -\frac{1}{216}(x-9)^2$

(b) $T_2(x) = 3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2$

(c) $T_2(x) = 3 + \frac{1}{6}(x-9) - \frac{1}{108}(x-9)^2$

(d) $T_2(x) = -\frac{1}{108}(x-9)^2$

(e) None of these.

9. The series $\sum_{n=1}^{\infty} ne^{-n^2}$

(a) converges by the Integral Test.

(b) diverges by the Ratio Test.

(c) diverges by the Test for Divergence.

(d) diverges by the p-series test.

(e) converges by the Comparison Test with $\sum_{n=1}^{\infty} e^{-n^2}$.

10. In three-dimensional space, R^3, the equation $x^2 + y^2 = 9$ describes

(a) a parabola.

(b) a sphere.

(c) a cylinder.

(d) a circle.

(e) a plane.

11. The series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(a) is absolutely convergent.

(b) is convergent but not absolutely convergent.

(c) is divergent.

(d) is absolutely divergent but not divergent.

(e) converges absolutely by the ratio test.

12. The series $\sum_{n=1}^{\infty} \frac{1}{e^n+\sqrt{n}}$ is

(a) divergent because $\frac{1}{e^n+\sqrt{n}} > \frac{1}{e^n}$ and $\sum_{n=1}^{\infty} \frac{1}{e^n}$ is a divergent geometric series.

(b) convergent because $\frac{1}{e^n+\sqrt{n}} < \frac{1}{e^n}$ and $\sum_{n=1}^{\infty} \frac{1}{e^n}$ is a convergent geometric series.

(c) convergent because $\frac{1}{e^n+\sqrt{n}} < \frac{1}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is a convergent p series.

(d) divergent because $\frac{1}{e^n+\sqrt{n}} < \frac{1}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p series.

(e) divergent because $\frac{1}{e^n+\sqrt{n}} > \frac{1}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p series.

13. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$.

(a) e^2

(b) $\cos(-2)$

(c) $\sin(-2)$

(d) 0

(e) e^{-2}

PART II: WORK OUT (48 points total)

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

14. a.) (5 pts) Using the known Maclaurin series for $\sin x$, find the Maclaurin series for $\sin(x^3)$.

b.) (5 pts) Using the Maclaurin series for $\sin(x^3)$, compute $\displaystyle\lim_{x \to 0} \frac{\sin(x^3) - x^3}{x^9}$.

15. (5 pts) Using the known Maclaurin series for e^x, find a Maclaurin series for e^{-x^2}.

b.) (5 pts) Expand the Maclaurin series for e^{-x^2} found above to degree 4. Use this 4^{th} degree Maclaurin polynomial to estimate $\int_0^1 e^{-x^2}\, dx$. Do not simplify.

16. (10 pts) Find a power series about 0 for $f(x) = \ln(6x + 5)$. What is the associated radius of convergence?

17. (10 pts) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n4^n}$. Be sure to test the endpoints for convergence.

18. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2+1}$.

a.) (4 pts) Prove the series is absolutely convergent.

b.) (2 pts) Find s_2, the sum of the first 2 terms, to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2+1}$.

c.) (2 pts) Find an upper bound on $|R_2|$, the absolute value of the remainder, in using s_2 to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2+1}$.

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Question	Points Awarded	Points
1-13		52
14		10
15		10
16		10
17		10
18		8
		100