MATH 152, Spring 2014
COMMON EXAM I - VERSION A

LAST NAME: Key FIRST NAME: ____________

INSTRUCTOR: ____________

SECTION NUMBER: ____________

UIN: ____________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. In Part 1 (Problems 1-12), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam!*
3. In Part 2 (Problems 13-17), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-12		48
13		8
14		10
15		8
16		10
17		16
		100

PART I: Multiple Choice: 4 points each

1. Find the area bounded by $y = \cos(2x)$, $y = \sin(2x)$, $x = 0$ and $x = \frac{\pi}{8}$.

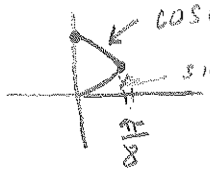
(a) $\sqrt{2} - 1$

(b) $2\sqrt{2} - 2$

(c) $\frac{1}{2} - \sqrt{2}$

(d) $\frac{\sqrt{2}}{2} - \frac{1}{2}$

(e) $\frac{1}{2}$



$$A = \int_0^{\frac{\pi}{8}} (\cos(2x) - \sin(2x))\,dx$$

$$= \left(\frac{1}{2}\sin(2x) + \frac{1}{2}\cos(2x)\right)\Big|_0^{\frac{\pi}{8}}$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}(0) + \frac{1}{2}(1)\right)$$

$$= \boxed{\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} - \frac{1}{2} = \frac{\sqrt{2}}{2} - \frac{1}{2}}$$

2. $\displaystyle\int_0^3 \frac{x}{\sqrt{x+1}}\,dx =$

(a) $\frac{17}{3}$

(b) $\frac{8}{3}$

(c) 12

(d) $\frac{20}{3}$

(e) $\frac{9}{2}$

$u = x + 1 <$ $u=4$, $u=1$

$du = dx$

$$\int_1^4 \frac{u-1}{\sqrt{u}}\,du = \int_1^4 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$$

$$= \left(\frac{2}{3}u^{\frac{3}{2}} - 2\sqrt{u}\right)\Big|_1^4$$

$$= \frac{2}{3}(8) - 2(2) - \left(\frac{2}{3} - 2\right)$$

$$= \frac{16}{3} - 4 - \frac{2}{3} + 2$$

$$= \frac{14}{3} - 2 = \boxed{\frac{8}{3}}$$

3. A 10 foot chain that weighs 20 pounds is used to lift a 30 pound weight. Assuming the chain is hanging from a ceiling, find the work done in pulling this chain and the attached weight to the top of the ceiling.

(a) 325 foot pounds

(b) 25 foot pounds

(c) 1300 foot pounds

(d) 400 foot pounds

(e) 600 foot pounds

$$W = \int_0^{10} 2x\,dx + 30(10)$$

$$= 100 + 300$$

$$= 400 \text{ ft-lbs}$$

4. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{x}{2}$ about the y-axis.

(a) $\frac{17\pi}{15}$

(b) $\frac{16\pi}{15}$

(c) $\frac{4\pi}{3}$

(d) $\frac{15\pi}{4}$

(e) $\frac{64\pi}{15}$

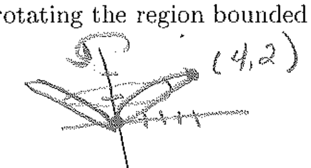
$\sqrt{x} = \frac{x}{2}$

$x = \frac{x^2}{4}$

$4x = x^2$

$x^2 - 4x = 0$

$x = 0,\ x = 4$



washers:

$$V = \int_0^2 \pi\left(4y^2 - y^4\right)dy$$

$$= \pi\left(\frac{4y^3}{3} - \frac{y^5}{5}\right)\Big|_0^2$$

$$= \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \pi\left(\frac{64}{15}\right)$$

5. Find the average value of $f(\theta) = \sec^2\left(\frac{\theta}{2}\right)$ over the interval $\left[0, \frac{\pi}{2}\right]$.

(a) $\frac{2}{\pi}$
(b) $\frac{1}{\pi}$
(c) $\frac{4}{\pi}$
(d) 2
(e) 0

$f_{ave} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{\theta}{2}\right) d\theta$

$= \frac{2}{\pi} \cdot 2\tan\frac{\theta}{2}\Big|_0^{\frac{\pi}{2}} = \frac{4}{\pi}(1-0)$

6. $\int_1^e x^3 \ln x \, dx =$

(a) $\frac{3e^4}{16} + \frac{1}{16}$
(b) $\frac{5e^4}{16} + \frac{1}{16}$
(c) $\frac{e^3}{4} - \frac{1}{4}$
(d) $\frac{e^4}{4} - \frac{1}{16}$
(e) $\frac{3e^4}{16} - \frac{1}{16}$

$u = \ln x \qquad dv = x^3 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^4}{4}$

$\int_1^e x^3 \ln x \, dx = \frac{x^4}{4} \ln x \Big|_1^e - \int_1^e \frac{1}{4} x^3$

$= \left(\frac{x^4}{4} \ln x - \frac{1}{16} x^4\right)\Big|_1^e$

$= \frac{e^4}{4} - \frac{e^4}{16} + \frac{1}{16} = \frac{3}{16} e^4 + \frac{1}{16}$

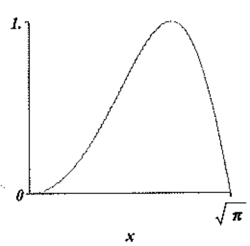
7. $\int \frac{x^2 + x - 1}{x^3} dx =$

(a) $\ln|x| - \frac{1}{x} + \frac{2}{x^2} + C$
(b) $\ln|x| - \frac{1}{x} + \frac{1}{2x^2} + C$
(c) $\ln|x| - \frac{2}{x^3} + \frac{3}{x^4} + C$
(d) $\ln|x| - \frac{1}{x} - \frac{1}{2x^2} + C$
(e) $\ln|x| + \frac{1}{x} + \frac{1}{2x^2} + C$

$\int \frac{x^2 + x - 1}{x^3} dx = \int \left(\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}\right) dx$

$= \ln|x| - \frac{1}{x} + \frac{1}{2x^2} + C$

8. Find the volume of the solid obtained by rotating the region bounded by $y = \sin(x^2)$, $y = 0$, $x = 0$, $x = \sqrt{\pi}$ about the y-axis.



shells: $V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2) \, dx$

$u = x^2$

$du = 2x \, dx$

$V = \pi \int_0^{\pi} \sin u \, du$

$= -\pi \cos u \Big|_0^{\pi}$

$= -\pi(-1-1) = 2\pi$

(a) π
(b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$
(d) 2π
(e) 4π

9. A spring has a natural length of 2 m. If a 30 N force is required to keep it stretched to a length of 4 m, how much work is required to stretch it from 3 m to 5 m?

(a) $\frac{290}{3}$ J
(b) 30 J
(c) 60 J
(d) 240 J
(e) 435 J

$f(x) = kx$

$30 = k(2)$

$k = 15$

$f(x) = 15x$

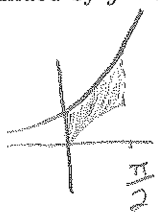
$W = \int_1^3 15x\,dx$

$= \frac{15}{2}x^2\Big|_1^3 = \frac{15}{2}(8)$

$= \boxed{60\text{ J}}$

10. Find the area bounded by $y = e^x$, $y = \sin x$, $x = 0$ and $x = \frac{\pi}{2}$.

(a) $e^{\pi/2} - 2$
(b) $e^{\pi/2}$
(c) $e^{\pi/2} + 2$
(d) $e^{\pi/2} - 1$
(e) $e^{\pi/2} + 1$



$A = \int_0^{\frac{\pi}{2}} (e^x - \sin x)\,dx$

$= (e^x + \cos x)\Big|_0^{\frac{\pi}{2}} = e^{\frac{\pi}{2}} + 0 - (1+1)$

$= e^{\pi/2} - 2$

11. If we revolve the region bounded by $y = \frac{1}{4}x^2$ and $y = 5 - x^2$ about the x-axis, which of the following integrals gives the resulting volume?

(a) $\int_{-2}^{2} 2\pi\left((5-x^2)^2 - \left(\frac{1}{4}x^2\right)^2\right)dx$

(b) $\int_{-2}^{2} \pi\left(\left(5 - \frac{5}{4}x^2\right)^2\right)dx$

(c) $\int_{-2}^{2} \pi\left((5-x^2)^2 - \left(\frac{1}{4}x^2\right)^2\right)dx$

(d) $\int_{-4}^{4} \pi\left((5-x^2)^2 - \left(\frac{1}{4}x^2\right)^2\right)dx$

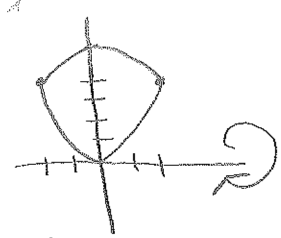
(e) $\int_{-1/2}^{1/2} \pi\left((5-x^2)^2 - \left(\frac{1}{4}x^2\right)^2\right)dx$

$\frac{1}{4}x^2 = 5 - x^2$

$x^2 = 20 - 4x^2$

$5x^2 = 20$

$x^2 = 4$



$V = \int_{-2}^{2} \pi\left((5-x^2)^2 - \left(\frac{1}{4}x^2\right)^2\right)dx$

12. If we revolve the region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$ about the line $y = 2$, which of the following integrals gives the resulting volume?

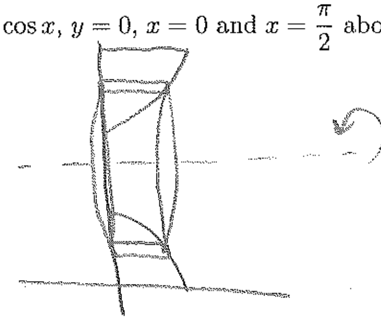
(a) $\int_0^{\pi/2} 2\pi(2-y)\arccos(y)\,dy$

(b) $\int_0^{1} 2\pi(y-2)\arccos(y)\,dy$

(c) $\int_0^{\pi/2} \pi(2 - \arccos(y))^2\,dy$

(d) $\int_0^{1} \pi(4 - (\arccos^2(y)))\,dy$

(e) $\int_0^{1} 2\pi(2-y)\arccos(y)\,dy$

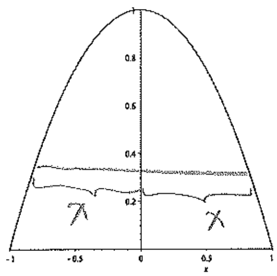


$V = \int_0^1 2\pi(2-y)\arccos y\,dy$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

13. (8 pts) Find the volume of the solid S described here: The base of S (drawn below) is the region bounded by $y = 1 - x^2$ and the x axis. Cross sections perpendicular to the y axis are squares.



$$V = \int_0^1 s^2\,dy$$

$$V = \int_0^1 \left(2\sqrt{1-y}\right)^2 dy$$

$$= 4\int_0^1 (1-y)\,dy$$

$$= 4\left(y - \tfrac{1}{2}y^2\right)\Big|_0^1$$

$$= 4\left(1 - \tfrac{1}{2}\right)$$

$$= 4\left(\tfrac{1}{2}\right) = \boxed{2}$$

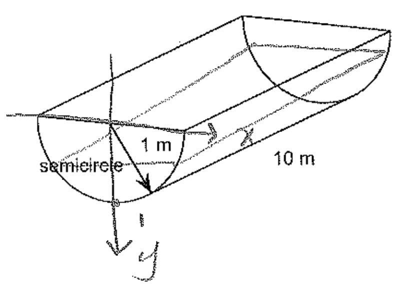
$$s = 2x = 2\sqrt{1-y}$$

$$y = 1 - x^2$$

$$x^2 = 1 - y$$

$$x = \sqrt{1-y}$$

14. (10 pts) A trough is 10 m long and is filled with water. The end of the trough is a semicircle with radius 1 m. Find the work done in pumping all of the water to the top of the tank. Note: The density of water is $\rho = 1000kg/m^3$ and the acceleration due to gravity is $g = 9.8m/s^2$.



$$V_s = (2x)(10)\Delta y$$

$$V_s = \left(2\sqrt{1-y^2}\right)(10)\Delta y$$

$$F_s = (20)(9800)\sqrt{1-y^2}\,\Delta y$$

$$W_s = (20)(9800)\,y\sqrt{1-y^2}\,\Delta y$$

$$x^2 + y^2 = 1$$

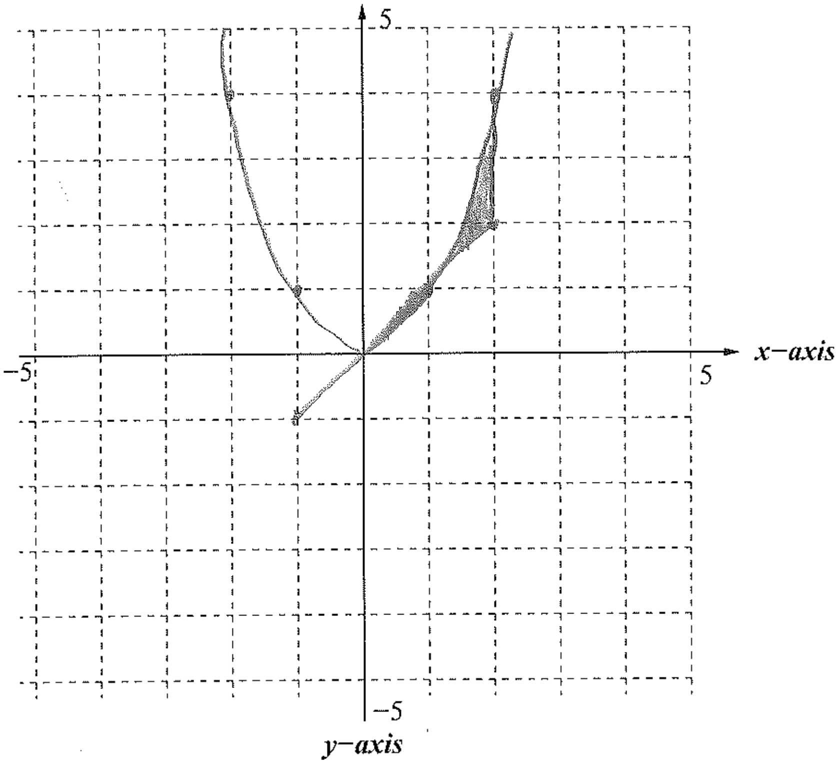
$$x = \sqrt{1-y^2}$$

$$W = 19600\int_0^1 y\sqrt{1-y^2}\,dy \qquad u = 1-y^2 \quad du = -2y\,dy$$

$$= \frac{19600}{-2}\int_1^0 u^{\frac{1}{2}}\,du = \frac{19600}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^1$$

$$= \boxed{19600/3\ \text{J}}$$

15. a.) (2 pts) Sketch the region bounded by $y = x^2$, $y = x$, $x = 0$ and $x = 2$. Clearly label all intersection points.



b.) (6 pts) Find the area of this region.

$$A = \int_0^1 (x - x^2)\,dx + \int_1^2 (x^2 - x)\,dx$$

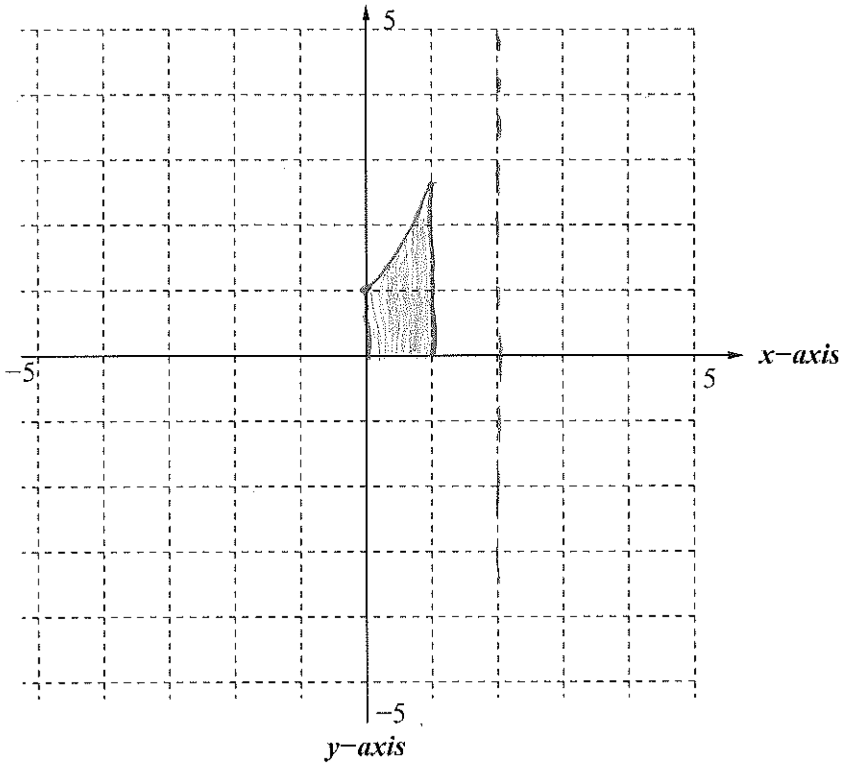
$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2}$$

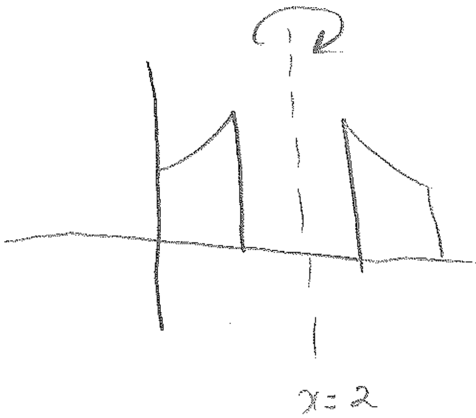
$$= -1 - \frac{2}{3} + \frac{8}{3}$$

$$= \frac{-3 - 2 + 8}{3} = 1$$

16. a.) (2 pts) Sketch the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 1$.



b.) (8 pts) Find the volume of the solid obtained by rotating this region about the line $x = 2$.



$$V = \int_0^1 2\pi(2-x)e^x\,dx$$

$u = 2 - x \qquad dv = e^x\,dx$

$du = -dx \qquad v = e^x$

$$\int (2-x)e^x\,dx = uv - \int v\,du$$
$$= (2-x)e^x - \int e^x(-dx)$$
$$= (2-x)e^x + e^x$$

$$2\pi\int_0^1 (2-x)e^x\,dx = 2\pi\left((2-x)e^x + e^x\right)\Big|_0^1 = 2\pi\left(e + e - (2+1)\right)$$
$$= \boxed{2\pi(2e-3)}$$

17. Integrate:

a.) (8 pts) $\int \arcsin(x)\,dx$

$u = \arcsin x \qquad dv = dx$

$du = \dfrac{1}{\sqrt{1-x^2}} \qquad v = x$

$\int \arcsin x \, dx = uv - \int v\,du$

$= x\arcsin x - \int \dfrac{x}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$\frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$= \sqrt{u}$

$= \sqrt{1-x^2}$

$= \boxed{x\arcsin x + \sqrt{1-x^2} + C}$

b.) (8 pts) $\int \frac{x+1}{x^2+1}\,dx = \int \dfrac{x}{x^2+1}\,dx + \int \dfrac{dx}{x^2+1}$

u-sub

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}|u|$

$= \frac{1}{2}\ln|x^2+1|$

arctan x

$\boxed{\frac{1}{2}\ln|x^2+1| + \arctan x + C}$