MATH 152, Spring 2014
COMMON EXAM II - VERSION A

LAST NAME: Key FIRST NAME: ____________________

INSTRUCTOR: ____________________

SECTION NUMBER: ____________

UIN: ____________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam!*
3. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		60
16		9
17		10
18		7
19		6
20		8
		100

PART I: Multiple Choice: 4 points each

1. Evaluate $\int_0^4 \frac{4x^2}{x+2}\,dx$.

   (a) $64 + 16\ln(3)$
   (b) $16\ln(12)$
   (c) $64 + 16\ln(12)$
   (d) $16\ln(3)$
   (e) Integral diverges

$$\begin{array}{r} 4x - 8 \\ x+2 \enclose{longdiv}{4x^2} \\ \underline{4x^2 + 8x} \\ -8x \\ \underline{-8x - 16} \\ 16 \end{array}$$

$$\int_0^4 \left(4x - 8 + \frac{16}{x+2}\right) dx$$

$$= \left(2x^2 - 8x + 16\ln|x+2|\right)\Big|_0^4 \to 16\ln 6 - 16\ln 2$$

$$= \boxed{16\ln 3}$$

2. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is given by $s_n = \frac{n}{2n+1}$, what is a_4?

   (a) $a_4 = \frac{1}{2}$
   (b) $a_4 = \frac{1}{63}$
   (c) $a_4 = \frac{55}{63}$
   (d) $a_4 = \frac{1}{99}$
   (e) $a_4 = \frac{89}{99}$

$$a_4 = S_4 - S_3$$

$$= \frac{4}{9} - \frac{3}{7}$$

$$= \frac{28 - 27}{63}$$

$$\boxed{a_4 = \frac{1}{63}}$$

3. Which of the following series diverges by the Test for Divergence?

   (a) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$
   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+2}$
   (c) $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$
   (d) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$
   (e) $\sum_{n=1}^{\infty} \frac{4^n}{9^{n+2}}$

$$\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos 0$$
$$= 1 \neq 0$$

$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$ diverges

4. After making the appropriate trigonometric substutition, the integral $\int_0^2 x^2\sqrt{4-x^2}\,dx$ becomes:

(a) $\int_0^{\pi/4} 16\tan^2\theta\sec^4\theta\,d\theta$

(b) $\int_0^{\pi/2} 8\sin^2\theta\cos\theta\,d\theta$

(c) $\int_0^{\pi/2} 8\sin^2\theta\cos^2\theta\,d\theta$

(d) $\int_0^{\pi/2} 16\sin^2\theta\cos^2\theta\,d\theta$

(e) $\int_0^{\pi/4} 8\tan^2\theta\sec^2\theta\,d\theta$

$x = 2\sin\theta$; $x=2,\ \theta=\frac{\pi}{2}$; $x=0,\ \theta=0$

$dx = 2\cos\theta\,d\theta$

$\int_0^{\frac{\pi}{2}} 4\sin^2\theta\sqrt{4-4\sin^2\theta}\,2\cos\theta\,d\theta$

$\int_0^{\frac{\pi}{2}} 4\sin^2\theta\cdot 2\cos\theta\cdot 2\cos\theta\,d\theta$

$\boxed{\int_0^{\frac{\pi}{2}} 16\sin^2\theta\cos^2\theta\,d\theta}$

5. The sequence $a_n = \ln(n) - \ln(4n+2)$

(a) converges to 0

(b) converges to $\ln(4)$

(c) converges to $\ln\left(\frac{1}{4}\right)$

(d) diverges to ∞

(e) diverges to $-\infty$

$\lim_{n\to\infty}\left[\ln(n) - \ln(4n+2)\right]$

$\lim_{n\to\infty}\ln\left(\frac{n}{4n+2}\right)$

$=\boxed{\ln\frac{1}{4}}$

6. Find the length of the curve $y = x^{3/2}$, $0 \le x \le 4$.

(a) $\frac{3}{2}(10\sqrt{10}-1)$

(b) $\frac{64}{27}$

(c) $\frac{2}{3}(10\sqrt{10}-1)$

(d) $\frac{8}{27}10\sqrt{10}$

(e) $\frac{8}{27}(10\sqrt{10}-1)$

$y' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

$u = 1+\frac{9}{4}x$; $u=10$; $u=1$

$L = \int_0^4\sqrt{1+\frac{9}{4}x}\,dx$ $\quad du = \frac{9}{4}dx$

$L = \frac{4}{9}\int_1^{10}u^{\frac{1}{2}}\,du = \frac{4}{9}\frac{2}{3}u^{3/2}\Big|_1^{10}$

$= \frac{8}{27}(10\sqrt{10}-1)$

7. $\int \tan^4 x \sec^4 x \, dx =$

(a) $\frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$

(b) $\frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + C$

(c) $-\frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$

(d) $\frac{1}{7}\tan^7 x - \frac{1}{5}\tan^5 x + C$

(e) $\frac{1}{5}\tan^5 x + \frac{1}{5}\sec^4 x + C$

$\int \tan^4 x \sec^2 x \sec^2 x \, dx$

$\int \tan^4 x(\tan^2 x + 1)\sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u^4(u^2+1)\,du = \int (u^6 + u^4)\,du$

$= \frac{1}{7}u^7 + \frac{1}{5}u^5 + C$

$= \frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$

8. $\sum_{n=1}^{\infty} \frac{3^n}{4^{n+1}} =$

(a) 1

(b) 0

(c) $\frac{3}{28}$

(d) 4

(e) $\frac{3}{4}$

$\sum_{n=1}^{\infty} \frac{3^n}{4 \cdot 4^n}$

$\sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{3}{4}\right)^n$

$\sum_{n=1}^{\infty} \frac{3}{16}\left(\frac{3}{4}\right)^{n-1} = \frac{\frac{3}{16}}{1-\frac{3}{4}} = \frac{\frac{3}{16}}{\frac{1}{4}} = \frac{3}{4}$

9. Which of the following integrals gives the surface area obtained by revolving the curve $y = \arctan(x)$, $0 \le x \le 1$, about the x-axis?

(a) $\int_0^1 2\pi x \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2}\, dx$

(b) $\int_0^1 2\pi \arctan(x) \sqrt{1 + \left(\frac{1}{x^2-1}\right)^2}\, dx$

(c) $\int_0^1 2\pi \arctan(x) \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2}\, dx$

(d) $\int_0^1 2\pi x \sqrt{1 + \left(\frac{1}{x^2-1}\right)^2}\, dx$

(e) $\int_0^1 2\pi \arctan(x) \sqrt{1 + \left(\frac{1}{\sqrt{x^2+1}}\right)^2}\, dx$

$y' = \frac{1}{1+x^2}$

$r = \arctan x$

$SA = \int_0^1 2\pi \arctan x \sqrt{1 + \left(\frac{1}{1+x^2}\right)^2}\, dx$

10. The integral $\int_{-1}^{2} \frac{dx}{x^2}$ $= \overset{①}{\int_{-1}^{0}} \frac{dx}{x^2} + \overset{②}{\int_{0}^{2}} \frac{dx}{x^2}$

(a) diverges to $-\infty$
(b) diverges to ∞
(c) diverges to 2
(d) diverges to $\frac{1}{2}$
(e) diverges to $\ln 4$

① $\lim_{t \to 0^-} \int_{-1}^{t} \frac{dx}{x^2} = \lim_{t \to 0^-} -\frac{1}{x}\Big|_{-1}^{t} = \lim_{t \to 0^-} \left(-\frac{1}{t} - 1\right) = \infty$

② $\lim_{t \to 0^+} \int_{t}^{2} \frac{dx}{x^2} = \lim_{t \to 0^+} -\frac{1}{x}\Big|_{t}^{2}$

$= \lim_{t \to 0^+} \left(-\frac{1}{2} + \frac{1}{t}\right) = \infty$

11. Which of the following sequences is both bounded and decreasing?

(a) $a_n = e^{-n}$
(b) $a_n = \cos n$
(c) $a_n = \ln n$
(d) $a_n = \left(\frac{-1}{2}\right)^n$
(e) $a_n = 1 - \frac{1}{n^2}$

$a_n = e^{-n} = \frac{1}{e^n}$

is bounded becouse $0 < \frac{1}{e^n} < \frac{1}{e}$

and decreases because

$a_{n+1} < a_n$

$\frac{1}{e^{n+1}} < \frac{1}{e^n}$

12. Find the length of the curve $x = \frac{t^2}{2}$, $y = \frac{t^3}{3}$, $0 \le t \le 1$.

(a) $\frac{5}{6}$
(b) $\frac{2}{3}(2\sqrt{2} - 1)$
(c) $\frac{3}{4}(2\sqrt{2} - 1)$
(d) $\frac{11}{6}$
(e) $\frac{1}{3}(2\sqrt{2} - 1)$

$\frac{dx}{dt} = t \qquad \frac{dy}{dt} = t^2$

$L = \int_0^1 \sqrt{t^2 + t^4}\, dt$

$= \int_0^1 \sqrt{t^2(1+t^2)}\, dt$

$u = 1 + t^2 \quad u=2, \ u=1$

$du = 2t\,dt$

$= \int_0^1 t\sqrt{1+t^2}\, dt$

$= \frac{1}{2}\int_1^2 u^{\frac{1}{2}}\, dt$

$\frac{1}{2}\,\frac{2}{3}u^{3/2}\Big|_1^2 = \frac{1}{3}(2\sqrt{2} - 1)$

13. $\int \sin^3 x \cos^2 x \, dx =$

(a) $\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

(b) $-\frac{1}{3}\cos^3 x - \frac{1}{4}\cos^4 x + C$

(c) $-\frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(d) $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

(e) $\left(\frac{1}{4}\sin^4 x\right)\left(\frac{1}{3}\cos^3 x\right) + C$

$\int \sin^2 x \cos^2 x \sin x \, dx$

$\int (1-\cos^2 x)\cos^2 x \sin x \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$-\int (1-u^2)u^2 \, du$

$-\int (u^2 - u^4) \, du \rightarrow -\frac{u^3}{3} + \frac{u^5}{5} + C$

$\boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$

14. $\int \frac{x+2}{x^2(x-1)} \, dx =$

(a) $-3\ln|x| + \frac{2}{x} + 3\ln|x-1| + C$

(b) $3\ln|x| + \frac{2}{x} + 3\ln|x-1| + C$

(c) $-\frac{2}{x} + 3\ln|x-1| + C$

(d) $-3\ln|x| - \frac{2}{x} + 3\ln|x-1| + C$

(e) $\frac{2}{x} + 3\ln|x-1| + C$

$\frac{x+2}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\rightarrow x+2 = Ax(x-1) + B(x-1) + Cx^2$

$x=0: \quad 2 = B(-1) \Rightarrow \boxed{B=-2}$

$x=1: \quad \boxed{3=C}$

$x+2 = Ax(x-1) - 2(x-1) + 3x^2$

$x=2: \quad 4 = A(2) - 2 + 12$

$4 = 2A + 10 \rightarrow \boxed{A=-3}$

$\int\left(\frac{-3}{x} - \frac{2}{x^2} + \frac{3}{x-1}\right) = -3\ln|x| + \frac{2}{x} + 3\ln|x-1| + C$

15. Find the surface area obtained by rotating the curve $y = x^2$, $0 \le x \le \sqrt{2}$, about the y-axis.

(a) $\frac{\pi}{6}(5\sqrt{5}-1)$

(b) $\frac{26\pi}{3}$

(c) $\frac{13\pi}{3}$

(d) $\frac{9\pi}{2}$

(e) $\frac{\pi}{8}(5\sqrt{5}-1)$

$y' = 2x$

$\int_0^{\sqrt{2}} 2\pi x\sqrt{1+4x^2} \, dx$

$u = 1+4x^2$

$du = 8x \, dx$

$\frac{\pi}{4}\int_1^9 u^{\frac{1}{2}} \, du \rightarrow \frac{\pi}{4}\frac{2}{3}u^{3/2}\Big|_1^9$

$\frac{\pi}{6}(27-1) = \frac{\pi}{6}(26)$

$= \frac{13\pi}{3}$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (9 pts) Integrate $\int \frac{4}{x^3+4x}\,dx$.

PFD: $\frac{4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$4 = A(x^2+4) + (Bx+C)(x)$

$x=0: \quad 4 = A(4) \quad \boxed{A=1}$

$4 = x^2+4+Bx^2+Cx$

$4 = (1+B)x^2 + Cx + 4$

$1+B=0 \quad \boxed{C=0}$

$\boxed{B=-1}$

$\int\left(\frac{1}{x} - \frac{x}{x^2+4}\right) = \ln|x| - \frac{1}{2}\ln(x^2+4) + C$

17. (a) (6 pts) Find s_n, the nth partial sum, for the series $\sum_{n=1}^{\infty}\left(\cos\left(\frac{1}{n}\right)-\cos\left(\frac{1}{n+1}\right)\right)$.

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = \cos(1) - \cos\left(\tfrac{1}{2}\right) + \cos\left(\tfrac{1}{2}\right) - \cos\left(\tfrac{1}{3}\right) + \dots + \cos\left(\tfrac{1}{n}\right) - \cos\tfrac{1}{n+1}$$

$$\boxed{S_n = \cos(1) - \cos\left(\tfrac{1}{n+1}\right)}$$

(b) (4 pts) Using the nth partial sum found in part (a), find the sum of the series $\sum_{n=1}^{\infty}\left(\cos\left(\frac{1}{n}\right)-\cos\left(\frac{1}{n+1}\right)\right)$.

$$\sum_{n=1}^{\infty}\left[\cos\left(\tfrac{1}{n}\right) - \cos\left(\tfrac{1}{n+1}\right)\right] = \lim_{n\to\infty}\left[\cos(1) - \cos\left(\tfrac{1}{n+1}\right)\right]$$

$$= \boxed{\cos(1) - 1}$$

18. (7 pts) Using a comparison theorem, prove that the integral $\int_1^{\infty}\frac{dx}{x+e^{4x}}$ converges.

$$\int_1^{\infty}\frac{dx}{x+e^{4x}} \le \int_1^{\infty}\frac{dx}{e^{4x}}$$

$$= \lim_{t\to\infty}\int_1^{t} e^{-4x}\,dx$$

$$= \lim_{t\to\infty} -\tfrac{1}{4}e^{-4x}\Big|_1^t$$

$$= \lim_{t\to\infty} -\tfrac{1}{4}\left(e^{-4t} - e^{-4}\right) = \boxed{\frac{1}{4e^4}}$$

larger integral converges so must smaller

19. (6 pts) Find $\int \sin^2 x \cos^2 x \, dx = \int \frac{1}{2}(1-\cos(2x)) \cdot \frac{1}{2}(1+\cos(2x))\, dx$

$$= \frac{1}{4}\int \left(1-\cos^2(2x)\right) dx$$

$$= \frac{1}{4}\int \sin^2(2x)\, dx$$

$$= \frac{1}{4}\int \frac{1}{2}\left(1-\cos(4x)\right) dx$$

$$= \frac{1}{8}\left(x - \frac{1}{4}\sin(4x)\right) + C$$

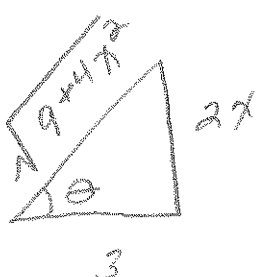
20. (8 pts) Find $\int \frac{dx}{\sqrt{4x^2+9}}$

$$2x = 3\tan\theta$$

$$x = \frac{3}{2}\tan\theta$$

$$dx = \frac{3}{2}\sec^2\theta \, d\theta$$

$$\int \frac{\frac{3}{2}\sec^2\theta \, d\theta}{\sqrt{9\tan^2\theta + 9}} = \frac{3}{2}\int \frac{\sec^2\theta \, d\theta}{3\sec\theta}$$



$$= \frac{1}{2}\int \sec\theta \, d\theta$$

$$= \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{9+4x^2}}{3} + \frac{2x}{3}\right| + C$$