Name_____

MATH 152H Exam 3 Spring 2017

Sections 203/204 (circle one) P. Yasskin

1-12	/60	14	/20
13	/ 7	15	/18
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. If it takes 12 Newtons of force to hold a spring at 3 meters from the rest position, how much work is done to stretch it from 2 meters to 4 meters from rest?

a. 24 Joules
b. 16 Joules
c. 8 Joules
d. 6 Joules
e. 4 Joules

2. Compute $\int_0^{\pi/2} \tan\theta \, d\theta$.

a. $-\infty$
b. -1
c. 0
d. 1
e. ∞

3. Compute $\int_4^{\infty} \frac{1}{x^{3/2}} \, dx$.

a. -1
b. 0
c. $\frac{1}{3}$
d. 1
e. ∞

4. Compute $\int_{-8}^{8} \frac{1}{x^{5/3}}\,dx$.

a. $-\infty$

b. ∞

c. divergent but not $\pm\infty$

d. 0

e. $\frac{3}{4}$

5. The differential equation $\frac{dy}{dx} = 2 + 2y + x + xy$ is

a. both separable and linear

b. separable but not linear

c. linear but not separable

d. neither separable nor linear

6. Find the integrating factor for the differential equation $x^3 \frac{dy}{dx} = x^5 + 3x^2y$.

a. $I = x^3$

b. $I = \frac{1}{x^3}$

c. $I = e^{3/x^2}$

d. $I = e^{-3/x^2}$

e. $I = e^{-x^3}$

7. Solve the initial value problem $\frac{dy}{dx} = \frac{4}{3}\frac{x^3}{y^2}$ with $y(1) = 2$. What is $y(0)$?

a. $\sqrt[3]{2}$

b. $\sqrt[3]{7}$

c. $-\sqrt[3]{15}$

d. 7

e. -15

8. Compute $\lim_{n\to\infty}\left(\frac{1}{n^4}\right)^{3/\ln n}$.

a. 12

b. 64

c. $e^{3/4}$

d. e^{12}

e. e^{-12}

9. Compute $\sum_{n=1}^{\infty}(-1)^n\frac{3}{2^n}$

a. -1

b. -3

c. 1

d. 2

e. 3

10. The series $\sum_{n=3}^{\infty} \frac{2n}{(n^2-4)^2}$

a. diverges by the n^{th} Term Divergence Test

b. converges by a Simple Comparison with $\sum_{n=3}^{\infty} \frac{2}{n^3}$

c. diverges by a Simple Comparison with $\sum_{n=3}^{\infty} \frac{2}{n^3}$

d. converges by the Integral Test

e. diverges by the Integral Test

11. If $S = \sum_{n=3}^{\infty} \frac{2n}{(n^2-4)^2}$ is approximated by its 12^{th}-partial sum $S_{12} = \sum_{n=3}^{12} \frac{2n}{(n^2-4)^2}$, then the error $E_{12} = S - S_{12}$ is less than

a. $\ln 140$

b. $\frac{1}{2}\ln 140$

c. $\left(\frac{1}{140}\right)^3$

d. $\left(\frac{1}{140}\right)^2$

e. $\frac{1}{140}$

12. Compute $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$. Note: $\frac{1}{n(n-1)} = \frac{n}{n-1} - \frac{n+1}{n}$

a. $\frac{1}{3}$

b. $\frac{1}{2}$

c. 1

d. 2

e. 3

Work Out: (Points indicated. Part credit possible. Show all work.)

13. $(7$ points$)$ A tank initially contains 25 L of salt water with a concentration of $40\ \frac{\text{gm of salt}}{\text{L}}$.

Salt water with concentration $10\ \frac{\text{gm of salt}}{\text{L}}$ is entering the tank at the rate $5\ \frac{\text{L}}{\text{hour}}$.

The water is kept well mixed and drains at $5\ \frac{\text{L}}{\text{hour}}$.

Set up the initial value problem for the amount of salt in the tank $S(t)$ at time t. Do not solve it.

a. Write the differential equation:

b. Write the initial condition:

14. $(20$ points$)$ A sequence is defined recursively by

$$a_{n+1} = \sqrt{a_n} + 6 \qquad \text{with} \qquad a_1 = 100$$

a. Assuming the limit $L = \lim_{n\to\infty} a_n$ exists, find the possible limits, L.

b. Write out the first 3 terms of the sequence:

c. State a conjecture about boundedness: (Circle one answer and fill in the blank.)

The sequence is bounded $\begin{matrix}\text{above}\\ \text{below}\end{matrix}$ by _____.

Now write the conjecture as an inequality:

d. Use Mathematical Induction to prove this conjecture.

(continued)

e. State a conjecture about monotonicity: (Circle one answer.)

The sequence is increasing / decreasing .

Now write the conjecture as an inequality:

f. Use Mathematical Induction to prove this conjecture.

g. What do you conclude about the convergence or divergence of the series? Name any theorem you use.

15. $(18$ points$)$ Solve the initial value problem:

$$\frac{dy}{dx} = 3x^5 - 3x^2y \qquad y(1) = 2e^{-1}$$

Give the explicit solution not just the implicit solution.